Notes on Electric Circuits

(Dr. Ramakant Srivastava)

Passive Sign Convention (PSC)

Passive sign convention deals with the designation of the polarity of the voltage and the direction of the current arrow in an element or a sub-circuit. The convention is satisfied when the current arrow is towards the terminal marked positive. The values of the voltage *v* and current *i* are irrelevant. For example, in Figure 1.1, PSC is satisfied for element B but not for element A.

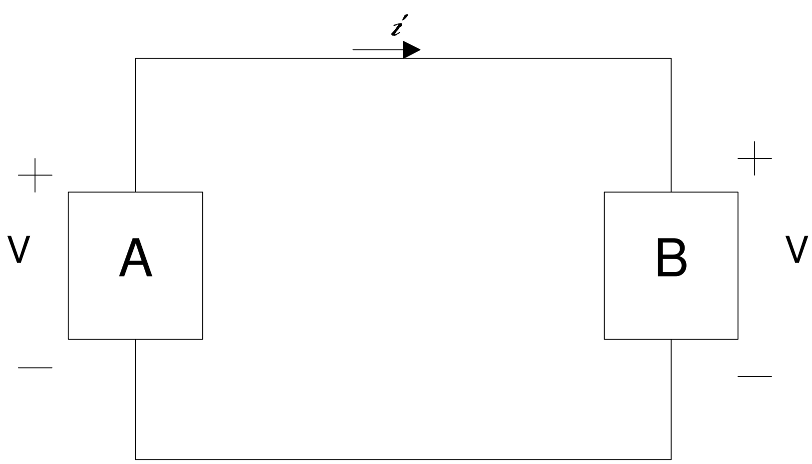


Figure 1.1: Passive sign convention (PSC)

Example 1.1
Consider the four equivalent circuits shown in Figure 1.2. Of the four, only the designations in (a) and (d) obey the convention and others do not.

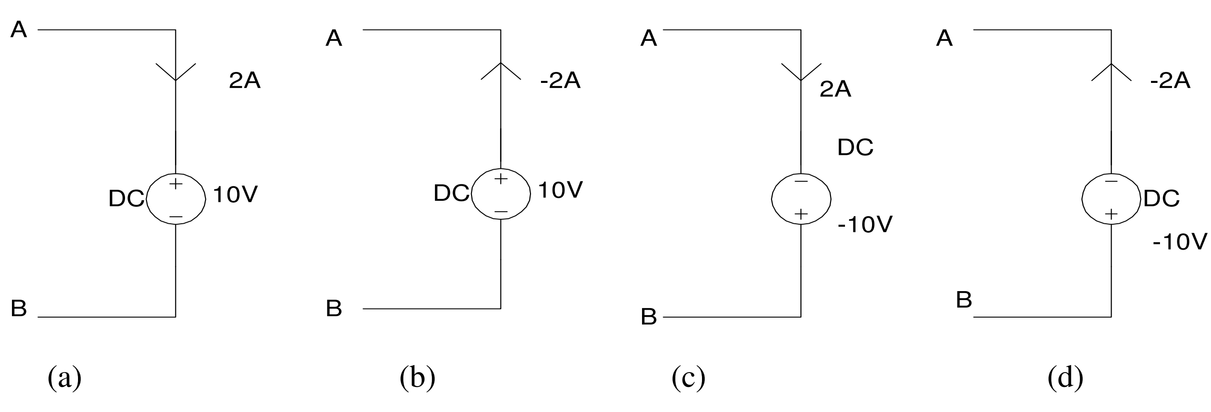


Figure 1.2: Equivalent representations of the same circuit

Exercise 1.1
Identify the sources in Figure 1.3 obeying the passive sign convention.

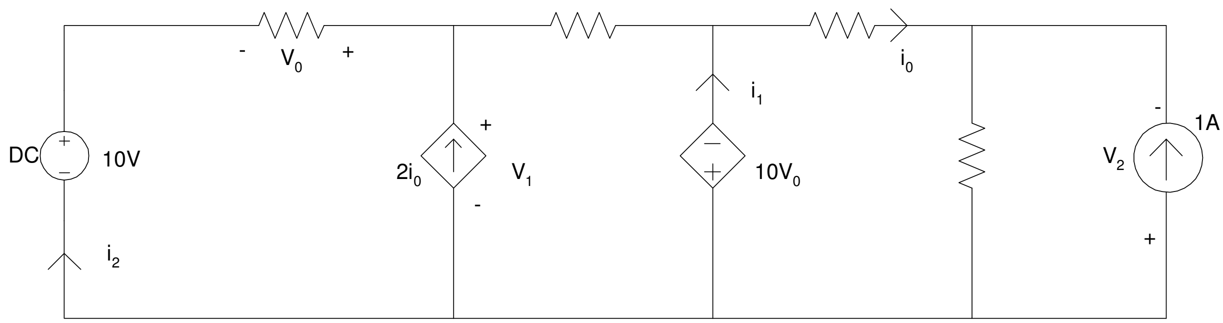


Figure 1.3: Identification of elements obeying the passive sign convention

Answer: The 1-A source and the dependent voltage source. Notice, we need not know the values of v_2, v_0 or i_1 to answer the question.

Power

If the voltage and current in an element (sub-circuit) satisfy PSC, then their product gives the value of the power absorbed by the element (sub-circuit). For example, in the circuit shown in Figure 1.1, $p = vi$ is the power absorbed by the element B. If p is positive, then the element (sub-circuit) absorbs power. On the other hand, a negative value for the absorbed power would imply positive power being provided.

Example 1.2
In the circuit of Figure 1.4, the resistor satisfies PSC and the product $vi = (10)(1) = 10$W. Therefore, the resistor absorbs 10 W of power. In the voltage source, PSC is not satisfied. However, if the switch the polarity of the source, then the convention will be satisfied and absorbed power = $(-10)(1) = -10$ W. Alternatively, we could keep the voltage polarity and switch the direction of the current arrow to satisfy PSO. Then the absorbed power = $(10)(-1) = -10$ W. The source is providing 10 W of power. Notice, the circuit is self sufficient in energy. Power conservation is satisfied in all electric circuits.

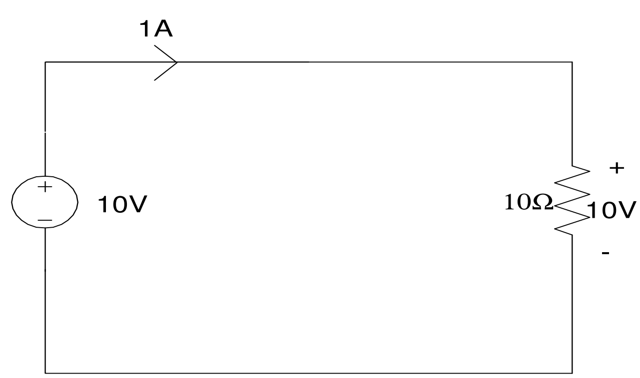


Figure 1.4: Passive sign convention and absorbed power

Example 1.3

The circuit of Figure 1.5 schematically represents the charging of a car battery. The 13-V source represents the battery charger. It provides 26 W. The 11-V source represents the car battery and it absorbs 22 W. The connecting cables are represented by a resistor. By KVL, there is a 2-V drop across the resistor. Thus it absorbs 4 W.

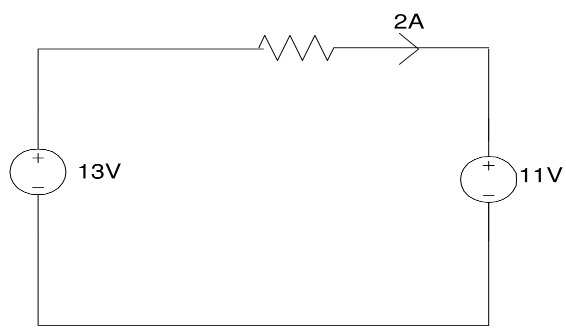


Figure 1.5: Schematics of a battery charging circuit

Example 1.4

In Figure 1.6, PSC is satisfied only by the dependent voltage source. It absorbs $20v_0$ of power. The two independent sources do not obey PSC. The current source provides 5 W and the 10- V source produces 10 W. By power conservation, the total power absorbed by the three resistors must be $(10 + 5 - 20v_0)$. Notice, that if v_0 is negative, then the dependent source provides positive power.

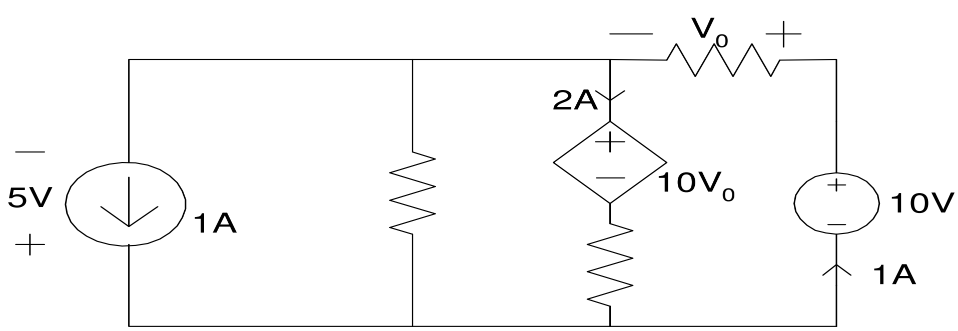
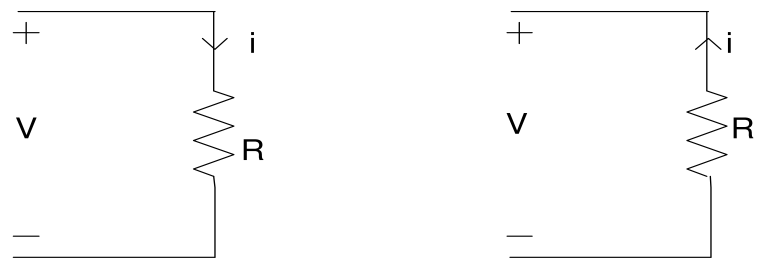


Figure 1.6: Power conservation

Resistors and Ohm's Law

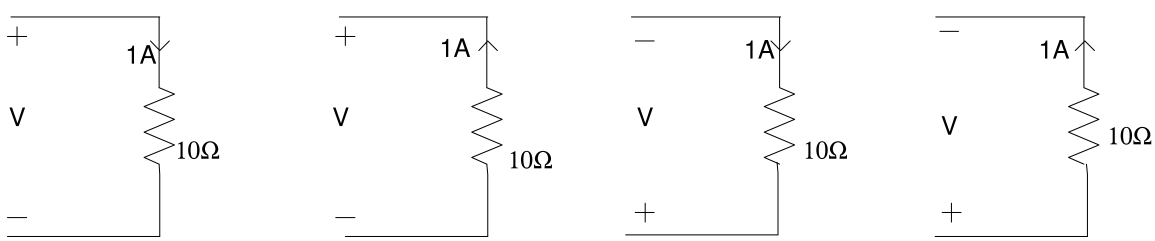
Figure 1.7(a) shows a resistor of resistance R where PSC is obeyed. According to Ohm's Law, $v = iR$. In Figure 1.7(b), PSC is not obeyed and Ohm's Law gives $v = -iR$.



(a) (b)

Figure 1.7: Ohm's Law and the passive sign convention

Example 1.5
We will show the application of Ohm's Law in the four circuits of Figure 1.8.
Figure 1.8(a): PSC obeyed. $v = (1)(10) = 10$ V.
Figure 1.8(b): PSC not obeyed. $v = (-1)(10) = -10$ V.
Figure 1.8 (c): PSC not obeyed. $v = (-1)(10) = -10$ V.
Figure 1.8 (d): PSC obeyed. $v = (1)(10) = 10$ V.



(a) (b) (c) (d)

Figure 1.8: Ohm's Law and the passive sign convention

Memory aid for Ohm's Law

One way to remember Ohm's Law correctly is to write in the following form:
$v_{ab} = i_{ab}R_{ab}$
In the above equation, v_{ab} is the voltage of terminal a with respect to terminal b and i_{ab} is the current flowing from a to b in the resistor R_{ab} connected to a and b. The equation implies that in a resistor, positive current always flows from the higher voltage terminal to the lower voltage terminal. If $v_{ab} < 0$, then $i_{ab} < 0$.

Short Circuit
There exists a short circuit between a and b if $R_{ab} = 0$. This results in $v_{ab} = 0$, independent of i_{ab}. All the current carrying perfect conductors are represented by short circuits and there is no voltage drop across any two points on such a conductor.

Example 1.6
In the circuit shown in Figure 1.9, the 20-Ω resistor is shorted since terminals a and b are connected by a zero-resistance resistor. Since $v_{ab} = 0$, there would be no current in the 20-Ω resistor as dictated by the Ohm's Law. 15-Ω and 10-Ω resistors are in parallel and the 0.4-A current in the 15-Ω resistor flows from a to b via the short circuit, by-passing the 20-Ω resistor.

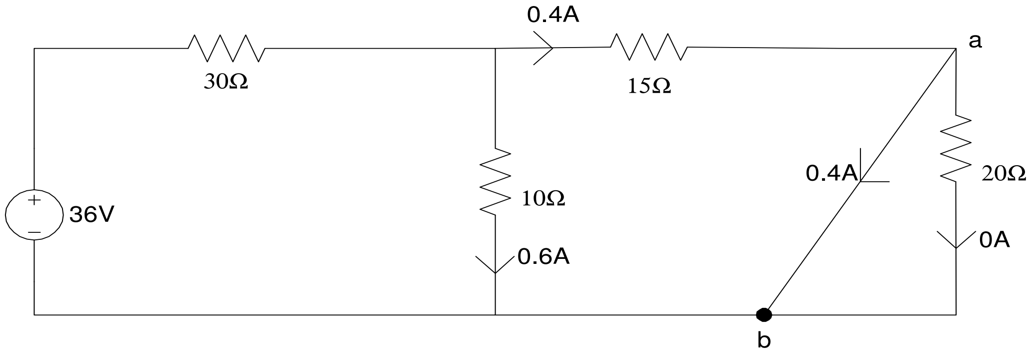


Figure 1.9: Circuit showing short circuit across a resistor

Power in resistors
Since $v = iR$ and $p = vi$ both are valid equations only when PSC is obeyed, the power absorbed by a resistor can be written as
$p = vi = v^2/R = i^2R$, showing that resistors always absorb power. The absorbed power is dissipated as heat. Resistive elements are therefore commonly used to provide heating. Coffee makers, cookers, irons, heaters, all are examples of resistive appliances.
Example 1.7
Consider the circuit in Figure 1.10 where all the voltages and currents are known. You can test their validity by verifying that all the three laws (KCL, KVL, and Ohm's Law) are satisfied everywhere.

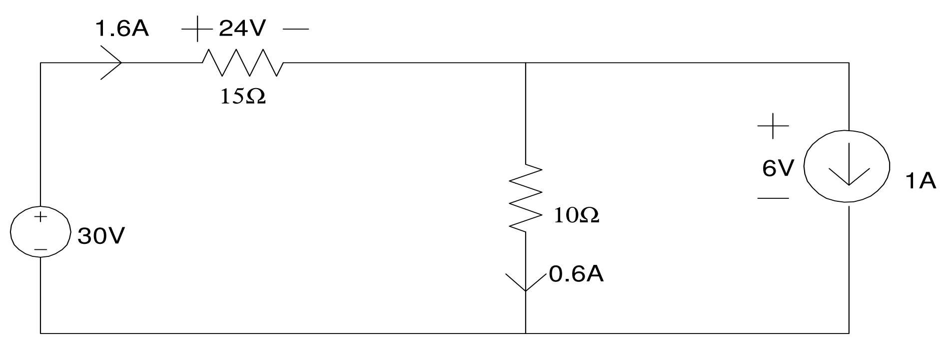


Figure 1.10: Power conservation

The powers absorbed by the various elements are as follows:
$P_{30V} = (-30)\ (1.6) = -48$ W,
$P_{1A} = (6)\ (1) = 6$ W,
$P_{15\Omega} = (24)\ (1.6) = (24)^2/15 = (1.6)^2\ (15) = 38.4$ W,
$P_{10\Omega} = (6)\ (0.6) = (6)^2/10 = (0.6)^2(10) = 3.6$ W.
It is obvious that the algebraic sum of all absorbed powers is zero, as required by the energy conservation law.

Series and parallel connections
Two elements are said to be in series if the same current passes through them.
Two elements are said to be in parallel if they share the same pair of nodes.
Two elements may neither be in series nor in parallel.

Example 1.8
In the circuit shown in Figure 1.11, the voltage source is in series with R_1 and the current source is in parallel with R_4. None of the three resistors is either in series or in parallel with any other resistor.

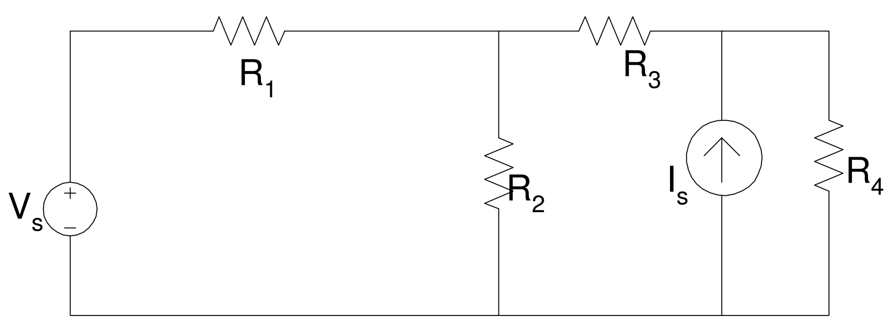


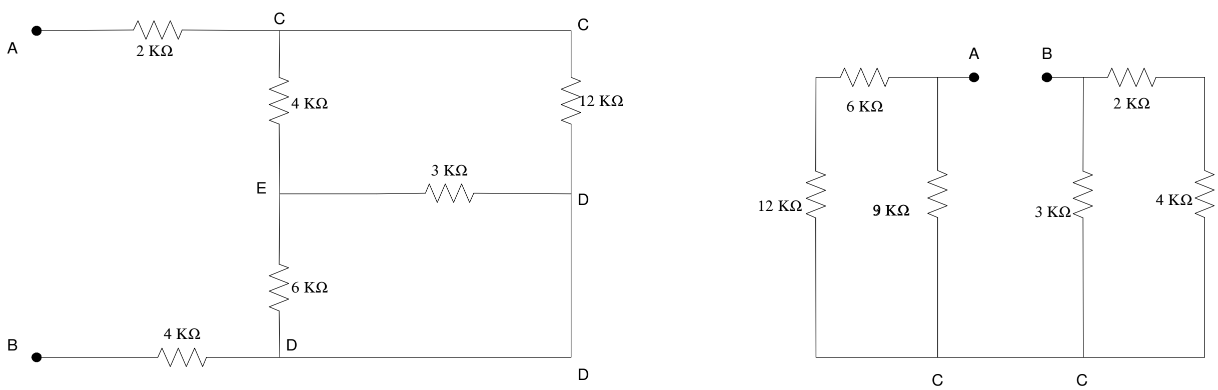
Figure 1.11: Series and parallel connections

Equivalent resistance
As seen from two terminals A and B, a network of resistors connected to A and B, is equivalent to a single resistor R_{AB}. Any source connected to A and B can not distinguish whether a network of several resistors is connected to the source or their equivalent resistance is connected to it.
There is a systematic procedure to find R_{AB} in circuits where resistors can be combined in a step-by-step fashion.
1) Connect a voltage or current source to A and B.
2) Identify all the nodes in the network. Remember, two points connected by a short circuit represent the same node and any resistor connected in parallel with a short circuit has no effect. Similarly, if only one terminal of a resistor is connected to the network, then that resistor has no effect.
3) Identify if any resistors are in parallel. If yes, combine them and replace them by their equivalent value, connecting the equivalent resistor to the same node pair where the original multiple resistors were connected in parallel.

4) Identify resistors in series. Combine them and replace them by their equivalent resistor between the terminals where they were originally connected. This step reduces the number of nodes in the network.
5) Repeat steps 3 and 4 until a single resistor R_{AB} is obtained.
Several examples are given here to demonstrate the procedure for finding R_{AB}.

Example 1.8(a): Figure 1.12(a)

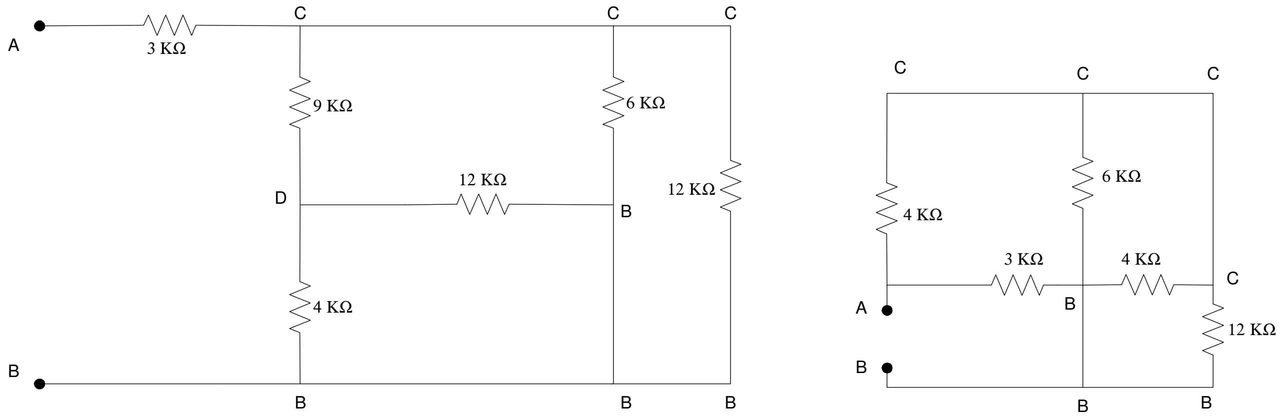
3 kΩ || 6 kΩ = 2 kΩ connected to E and D.
Between C and D, 4-kΩ and 2 kΩ in series, giving 6 kΩ, connected to C and D.
Node E is eliminated.
12 kΩ || 6 kΩ = 4 kΩ, connected to C and D.
R_{AB} = (2 + 4 + 4) kΩ = 10 kΩ.



Figures 1.12: Equivalent resistance: (a) (b)

Example 1.8(b): Figure 1.12(b)
6 kΩ + 12 kΩ = 18 kΩ.
18 kΩ||9 kΩ = 6 kΩ, connected to A and C.
(4 + 2) kΩ||3 kΩ = 2 kΩ, connected to C and B
R_{AB} = (6 + 2) kΩ = 8 kΩ.

Example 1.8(c): Figure 1.12(c)
4 kΩ||12 kΩ = 3 kΩ, connected to D and B.
Between C and B, 9 kΩ in series with 3 kΩ, giving 12 kΩ, connected to C and B. Node D is eliminated.
12 kΩ ||6 kΩ ||12 kΩ = 3 kΩ, connected to C and B.
R_{AB} = (3 + 3) kΩ = 6 kΩ.



Figures 1.12 : Equivalent resistance: (c) (d)

Example 1.8(d): Figure 1.12(d)
Between nodes B and C, 4 kΩ || 6 kΩ || 12 kΩ = 2 kΩ.
R_{AB} = (4 + 2) kΩ || 3 kΩ = 2 kΩ.

Example 1.8(e): Figure 1.12(e)
One of the 6-kΩ resistors is shorted.
One 6-kΩ resistor and the 3 kΩ resistor are in parallel, giving 2 kΩ between A and C.
R_{AB} = (2 + 4) kΩ || 6 kΩ = 3 kΩ.

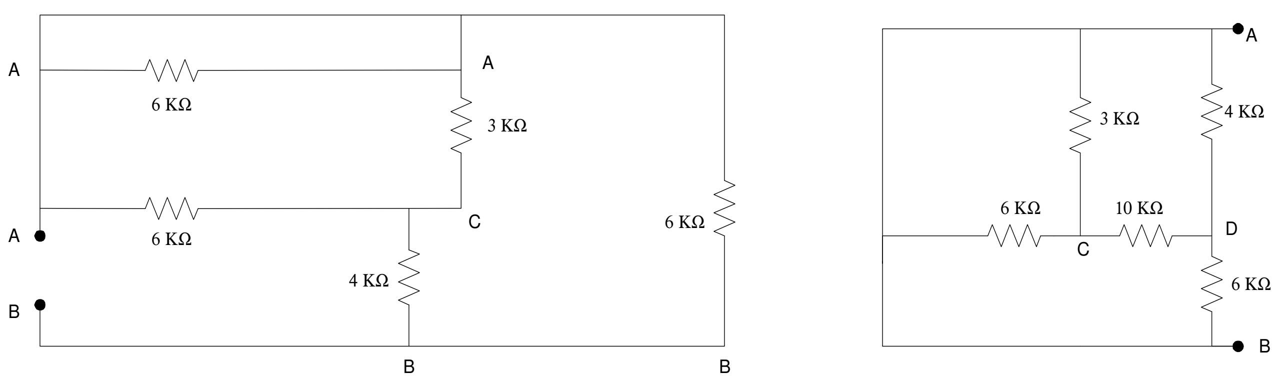


Figure 1.12: Equivalent resistance: (e) (f)

Example 1.8(f): Figure 1.12(f)
A and B are shorted. $R_{AB} = 0$.

The step-by-step method outlined above is not applicable to circuits where none of the resistors are in series or parallel to start with. In such cases, Delta-Wye or Wye-Delta transformations would be used first to apply the method.

Common Errors in Circuit Analysis

In application of current divider formula
* Wrong value of the equivalent resistance used.
* Formula applied to an inapplicable case (parallel elements incorrectly identified).
* Incorrect current division formula used.

In nodal analysis
* KCL is written incorrectly (mostly because Ohm's Law is applied incorrectly).
* KCL is written at a single node shared by a voltage source. Remember, if the voltage source is part of a supernode, you must write KCL for the supernode (both the terminals of the source simultaneously). If the voltage source is not part of a supernode, then the voltage of the non-reference terminal is already known and you need not write KCL for the non-reference node.
* Dependent sources are not given the same respect as independent sources.
* Nodes are incorrectly identified.
* Supernodes are not identified correctly.
* The number of independent equations does not correspond to the number of unknowns.
* Node voltages assigned without the reference node being shown.

In mesh analysis
* KVL is written incorrectly (mostly because Ohm's Law is applied incorrectly).
* Supermeshes are not correctly identified.
* Voltage across dependent sources is ignored.
* The number of independent equations does not correspond to the number of unknowns.
* Mesh currents not shown.

In source transformations
* The transformed pair is incorrectly placed.
* The polarity of the voltage source is inconsistent with arrow of current source.
* Current sources in parallel not added correctly.
* Voltage sources in series not added correctly.
* An element of interest is disturbed by inclusion in source transformation.

In finding Thevenin's equivalent circuit
* Open circuit not created (the load left connected).
* Equivalent resistance incorrectly calculated when no dependent sources present.
* Incorrect procedure used for equivalent resistance when dependent sources present.

In ideal op amp circuits
KCL applied to the output terminal to find its voltage.

In *i-v* characteristics of energy-storage elements
* Incorrect analytical expression for current or voltage waveform shown in the figure.
* Initial value ignored even when it was non-zero.
* Values of the given quantities not converted to proper units.
* Incorrect limits in the definite integral term.

* Forgetting which quantities must be continuous in an energy storage element.

In first-order transients
* Incorrect steady state behavior assumed for the energy storage element.
* Switching sequence interpreted incorrectly.
* Not following the step-by-step method correctly.
* Not focusing initially on the inductor current or capacitor voltage.
* Committing error in finding the Thevenin's resistance.
* Incorrect formula for the time constant.
* Use of incorrect formula for the first-order transient.
* Error in derivative of exponential functions.

In sinusoids, phasors and impedance
Failure to remember that ωt has units of radians.
Incorrect quadrant identification for the angle of a complex number in polar form.
Confusing absolute value of a complex number with its real part.
Confusing the angular frequency ω with the frequency f.
Failure to convert a sinusoid to the standard cosine-based form.
Mixing time domain quantities with frequency domain quantities.
Equating a phasor to its sinusoid.
Forgetting that impedance is not a phasor.
Adding phasors belonging to sinusoids of different frequencies.

Some Fundamentals of Electric Circuits

Current: Rate of flow of charge. Always shown by an arrow and algebraic value in units of Amperes (A).

Voltage: Work per unit charge. Always specified between two terminals and shown by polarity and algebraic value. Units are Volts (V). Alternatively shown by assigning two subscripts, the latter subscript being the reference. For example, V_{ab} is the voltage of terminal a with respect to terminal b. If a common reference has already been chosen, then $V_{ab} = V_a - V_b$; otherwise V_a and V_b have no meaning by themselves.

Node: Junction of two or more elements.

KCL: Algebraic sum of all currents reaching (or leaving) a node is zero. (Conservation of charge)

KVL: Algebraic sum of all voltages around a closed path is zero. (Conservation of energy)

Ohm's Law: Current I_{ab} flowing from terminal a to terminal b of a resistor of resistance R is given by $I_{ab} = V_{ab}/R$. Note, if V_{ab} is negative, I_{ab} will also be negative and vice versa.

Short Circuit: Terminals A and B are short circuited if $R_{AB} = 0$. Thus $V_{AB} = 0$.

Open Circuit: Terminals A and B have open circuit between them if $R_{AB} = \infty$. Thus $I_{AB} = 0$

Passive Sign Convention: Current arrow pointed towards the terminal where the voltage is marked positive.

Power Absorbed: Product of the voltage and current if passive sign convention has been observed. A negative value means positive power is being supplied.

Elements in Series: Two elements are said to be connected in series if the same current flows though them. If two currents in a circuit are equal, they are not necessarily the same.

Elements in Parallel: Two elements are said to be connected in parallel if they share the same pair of nodes. Since they are connected to the same two nodes, they will necessarily have the same voltage across them.

Voltage Divider: If several resistors are connected in series and the end-to-end voltage is V_0, then the current in each element is V_0/R_{eq}, where R_{eq} is the sum of all the resistances. The voltage across the resistor R_j is R_jV_0/R_{eq}.

Current Divider: If several resistors are in parallel and the total current entering one of the two nodes is I_t, then the voltage across the two nodes is I_tR_{eq} where R_{eq} is the reciprocal of the sum of the reciprocals of all the resistances. The current in resistor R_j is I_tR_{eq}/R_j.

Principle of Superposition: In a linear circuit containing multiple sources, the voltage or current at any location is the algebraic sum of the contributions of individual sources considered one at a time by deactivating other sources. A voltage source is deactivated when replaced by a short circuit and a current source when replaced by an open circuit.

Thevenin's Theorem: A linear circuit, as seen from two terminals, is equivalent to a voltage source V_T in series with a resistor R_T connected to the two terminals. V_T is the open-circuit voltage between the terminals. R_T is the equivalent resistance between the terminals when the sources have been deactivated.

Norton's Theorem: A linear circuit, as seen from two terminals, is equivalent to a current source I_N in parallel with a resistor R_T connected to the two terminals. I_N is the short-circuit current between the terminals. R_T is the equivalent resistance between the terminals when the sources have been deactivated.

Source Transformations: A voltage source V_s in series with a resistor R_s connected to a pair of nodes is equivalent to a current source I_s in parallel with the same resistor connected to the same pair of nodes. $V_s = R_sI_s$. The arrow of the current source points towards the terminals marked positive in the voltage source. The transformation is bi-directional.

Maximum Power Transfer: Maximum power available from two terminals of a linear network, is given by $V_T^2/4R_T$ where V_T and R_T represent the Thevenin's voltage and current seen from the two terminals. The condition for maximum power transfer requires that the load resistance be equal to R_T.

Some Conceptual Questions in Circuit Analysis

Sources, Resistors, KCL, KVL and Ohm's Law

- Is the effective resistance of N resistors in parallel always smaller than the smallest of the N resistors?
- If two resistors are not in parallel, do they have to be in series and vice versa?
- Is voltage across an open circuit always zero?
- Is current in a short circuit always zero?
- KCL implies conservation of which quantity?
- KVL implies conservation of which quantity?
- In a resistor, does a positive current always flow from the higher voltage terminal to the lower voltage terminal?
- Is resistor a linear element?
- What is an ideal voltage source and what are its *i*-*v* characteristics?
- What is an ideal current source and what are its *i*-*v* characteristics?
- Is the instantaneous power absorbed by a resistor always positive?
- Define a Volt in terms of work and charge.
- Define an Ampere in terms of charge and time.
- State the criterion for two elements to be in series.
- When are two elements in parallel?
- What is the significance of equivalent resistance?
- Can you add two voltage sources in series to get an equivalent source?
- Can you have two voltage sources of different value in parallel?
- Can you add two current sources in parallel to get an equivalent source?
- Can you have two current sources of different values in series?

Network Theorems

- What is the significance of Thevenin's voltage?
- What is the significance of Norton's current?
- State Thevenin's theorem in words.
- State Norton's theorem in words.
- What is meant by a linear circuit?
- What is the significance of Thevenin's resistance?
- Describe a method for measuring Thevenin's voltage and Thevenin's resistance. You are given a voltmeter, two resistors and copper wires.
- What is the condition for maximum power transfer from a network to a load?
- What is the expression for the maximum power available from a network?
- State the principle of superposition.
- When can you transform a voltage source and a resistor and into what?

Inductors, Capacitors, transients, and Sinusoids

- How does a capacitor behave in dc steady state?
- How does an inductor behave in dc steady state?
- What is meant by dc steady state?

- Does the current in a capacitor have to be continuous in time? Justify your answer.
- What must be continuous in time in an inductor, voltage or current? Why?
- Can the instantaneous power absorbed by an energy storage element be a negative quantity?
- What is meant by RMS value? Describe briefly in words.
- Can RMS value be defined for non-periodic voltage or current waveforms?
- What is the significance of the RMS value in relation to power absorbed by a resistor?
- In a first-order transient, does the capacitor current always decrease exponentially?
- In a first-order transient, does the inductor voltage always increase exponentially?
- When does one observe a first-order transient? Describe in a brief statement.
- In a few words, explain how a perfect square pulse gets distorted in passing through an R-C circuit.
- How does the R-C induced pulse distortion affect digital communications?

Phasors, Impedance, and Resonance

- Does a phasor exist for any periodic voltage or current?
- Can you define phase for any periodic waveform?
- Is impedance a phasor?
- Is phasor always a complex number?
- What is the difference between a phasor and a sinusoid, if any?
- Can phasors belonging to different frequencies be added?
- Does a unit less phasor exist?
- What is meant by inductive impedance?
- What is a phasor diagram? What do the two axes represent in the diagram?
- What is impedance triangle? Describe in few words.
- Can you always reduce equivalent impedance of a circuit to a rectangular form?
- Can the angle of the impedance (in polar form) be called the phase angle?
- What are the possible quadrants in which the angle of impedance can lie?
- How does a series combination of an inductor and a capacitor behave at the resonance frequency?
- How does a parallel combination of an inductor and a capacitor behave at the resonance frequency?
- Can an inductor in series with a resistor, be represented by an equivalent resistor in parallel with a capacitor?
- How can you use Wheatstone bridge to measure inductance or capacitance?

Frequency Response and Filters

- In a low-pass filter, is the dB value at the cut-off frequency always negative 3 dB?
- How would you define the cut-off frequency of a filter?

- What component in an R-L-C series band-pass filter is responsible for the resonance width?
- What do you understand by bandwidth? Explain briefly.
- In the Bode plot, what is plotted on the two axes?
- Where would you use a notch filter?
- What is an asymptotic Bode plot? What is the maximum error in such a plot?
- Why is Bode plot useful?

AC Power

- What is the significance of power factor?
- What is the significance of reactive power?
- What is the average power in an energy storage element?
- Of the four different kinds of power (average, reactive, complex, and apparent), which ones are always real? Give units for each.
- What is the power factor of a resistor?
- What do you understand by power triangle?
- What is the relationship between a phasor and its RMS phasor?
- Which one can be a complex number, RMS phasor or RMS value?
- Is the complex power a phasor?
- What is the relationship between apparent power and average power? When are they equal in magnitude?
- When do the reactive power and complex power have equal magnitude?
- Why is it desirable to have as large a power factor as possible?
- How would you increase the power factor of a capacitive load?
- List at least two applications of transformers.
- If primary of a transformer has fewer turns than the secondary, is the secondary current larger than the primary current or smaller?
- What is the significance of reflected load impedance?
- What is an ideal transformer?
- What do the two dots represent in the transformer symbol?
- Can you use transformers for DC voltages? Justify your answer.
- How many phases are present in a regular household?
- List at least two advantages of three-phase power.
- What is the meaning of balanced phases?
- Show in a phasor diagram three balanced phase voltages.
- If three balanced phases are connected in a Wye, which configuration of balanced load will deliver more power into the load, Wye or Delta? By what factor?
- How much current flows in the neutral wire in the Y-Y connection if the loads and phases are balanced.

Diodes and Rectifiers

- Show i-v characteristics of an ideal diode.
- Show i-v characteristics of an offset diode.

- Show i-v characteristics of a practical diode.
- What is the meaning of DC voltage in a periodic voltage waveform.
- What is the meaning of ripple in a rectified output voltage?
- How do you reduce the ripple?
- When you substantially reduce the ripple, by what factor does the DC voltage increase in a full-wave rectifier?

Some Useful Trigonometric Identities

$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\sin 2A = 2\sin A \sin B$
$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$
$\sin A = \cos(A - \pi/2)$
$\cos A = -\cos(A \pm \pi)$
$\sin A = -\sin(A \pm \pi)$

Some Useful Identities in Complex numbers

$j = \sqrt{(-1)}$
$j^2 = -1$
$j^3 = -j$
$j^4 = 1$
$1/j = -j$

$\exp(j\theta) = \cos\theta + j\sin\theta$ (Euler's identity)

$a + jb = \sqrt{(a^2 + b^2)} \exp(j\tan^{-1} b/a)$, the quadrant of $(\tan^{-1} b/a)$ depends on signs of a and b.
$\exp(\pm j\pi/2) = \pm j$
$\exp(\pm j\pi) = -1$
Complex conjugate of $(a + jb) = (a + jb)^* = (a - jb) = \sqrt{(a^2 + b^2)} \exp(-j\tan^{-1} b/a)$

Some Useful Relations in AC Power Calculations

Notation: All bold-face letters represent complex numbers.

Phasors

$\mathbf{X} = X_p \angle\theta_x$ (Complex quantity)

RMS Value of a Sinusoid

$X_{rms} = X_p / \sqrt{2}$ (Real quantity)

RMS Phasors

$\mathbf{V}_{rms} = V_{rms}\angle\theta_v$ (Complex quantity)

$\mathbf{I}_{rms} = I_{rms}\angle\theta_I$ (Complex quantity)

Impedance

$\mathbf{Z} = R + j\,X = |\mathbf{Z}|\angle\theta$ (Ω) (Complex quantity)

$|\mathbf{Z}| = (R^2 + X^2)^{1/2}$ (Real quantity)

$\theta = \tan^{-1}(X/R)$

Note: θ is the angle of the load impedance (We have suppressed the subscript z.)

Ohm's Law in Frequency Domain

$\mathbf{V} = \mathbf{IZ}$ (V) (Complex quantity)

$V_p = I_p\,|\mathbf{Z}|$ (All the quantities in this equation are real)

$V_{rms} = I_{rms}\,|\mathbf{Z}|$ (All the quantities in this equation are real)

$\theta_v - \theta_i = \theta$

$\theta > 0$ when $X > 0$ (Inductive impedance)

$\theta < 0$ when $X < 0$ (Capacitive impedance)

Average (Real) Power (W)

$P = V_{rms}\,I_{rms}\cos\theta = I_{rms}^2\,R = (V_{rms}^2\cos\theta)/|\mathbf{Z}|$ (Real quantity)

Power Factor (Real quantity)

$pf = \cos\theta = R/(R^2 + X^2)^{1/2}$, $1 \geq pf \geq 0$.

If $\theta > 0$ (inductive impedance), $\theta_i < \theta_v$, pf lagging

If $\theta < 0$ (capacitive impedance), $\theta_i > \theta_v$, pf leading

Reactive Power (VAR)

$Q = V_{rms}\,I_{rms}\sin\theta = I_{rms}^2\,X$ (Real quantity)

Apparent Power

$S = V_{rms}\,I_{rms} = I_{rms}^2\,|\mathbf{Z}| = V_{rms}^2/|\mathbf{Z}|$ (VA) (Real quantity)

Complex Power (VA)

$\mathbf{S} = V_{rms}\,I_{rms}\angle\theta = \mathbf{V}_{rms}\,\mathbf{I}_{rms}^* = P + j\,Q = I_{rms}^2\,\mathbf{Z} = V_{rms}^2/\mathbf{Z}^*$ (Complex quantity)

Step-by-Step Approach for First-Order Transients

Objective: Switching occurs at $t = 0$. Find the capacitor voltage $v_C(t)$ or inductor current $i_L(t)$ for $t > 0$.

Step 1. Find $v_C(0)$ or $i_L(0)$ in case they are not given. If the circuit was in a dc steady state at $t = 0^-$, then draw the circuit with the switch in the previous position, replacing the capacitor by an open circuit and inductor by a short circuit. Solve for $v_C(0^-)$ or

$i_L(0^-)$. Since capacitor voltage and inductor current are continuous functions of time, $v_C(0^+) = v_C(0^-)$ and $i_L(0^+) = i_L(0^-)$.

Step 2. Draw the circuit at $t > 0$ with the switch in the new position. Zero all the independent sources and find the Thevenin's resistance R_{Th} seen by the energy storage element.
Time constant, τ, is given by $R_{Th}C$ or L/R_{Th}.

Step 3. Draw the circuit at $t = \infty$. There is a new dc steady state with the switch is in the new position when the capacitor (inductor) acts as open (short) circuit. Solve for $v_C(\infty)$ or $i_L(\infty)$.

Step 4. Write the equation for the transient as:

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}e^{-t/\tau}, \qquad t > 0 \qquad (1)$$

where $x(t)$ represents $v_C(t)$ or $i_L(t)$.

Note 1: If the desired quantity is not the capacitor voltage or inductor current, but transient for voltage or current elsewhere in the circuit, then use the above results for the energy storage element to solve for the desired quantities.

Note 2. If the position of the switch changes again at $t = t_0$ and the objective is to find $x_1(t)$ for $t > t_0$, then use equation (1) to find $x(t_0)$. This will be the new initial value. In other words, $x_1(0) = x(t_0)$.
Now find $x_1(\infty)$ and new time constant τ_1 with the switch in the position at $t > t_0$.
The equation for the transient for the interval $t > t_0$ is given by

$$x_1(t) = x_1(\infty) + \{x_1(0) - x_1(\infty)\}\exp\{-(t-t_0)/\tau_1\} \qquad (2)$$

Note that the presence of $(t-t_0)$ in the exponential term guarantees that expression (2) satisfies the initial condition (at $t = t_0$).