

Lab 3: Theorems and the Wheatstone Bridge

OBJECTIVES

- To reinforce and verify the concepts behind the Superposition, Thévenin's, and Norton's Theorems
- To introduce and understand the concepts behind maximum power transfer
- To understand the design and use of a Wheatstone bridge circuit

MATERIALS

- The lab assignment (this document).
- Your lab parts.
- Pre-lab questions including Multisim.
- Graph paper.

INTRODUCTION

The Superposition Theorem

A statement of the superposition theorem is that the total effect of independent sources in a linear circuit is the algebraic sum of the effects of the independent sources considered separately. In the application of this theorem, the effect of only one independent source is considered at a time. All other independent sources are eliminated by replacing them with short circuits if they are voltage sources and with open circuits if they are current sources.

Thévenin's and Norton's Theorems

The application of either Thévenin's or Norton's theorem allows the reduction of a complex two-terminal circuit into a simple equivalent circuit that has the same output characteristics as the original circuit; i.e., a *black box*. Thévenin's theorem states that any linear two-terminal circuit, as in Fig. 1a, may be replaced by a Thévenin voltage source of voltage V_{Th} in series with a Thévenin resistance R_{Th} , as shown in Fig. 1b. This equivalent circuit will produce the same voltages and currents in any *external circuit* as does the original linear circuit.

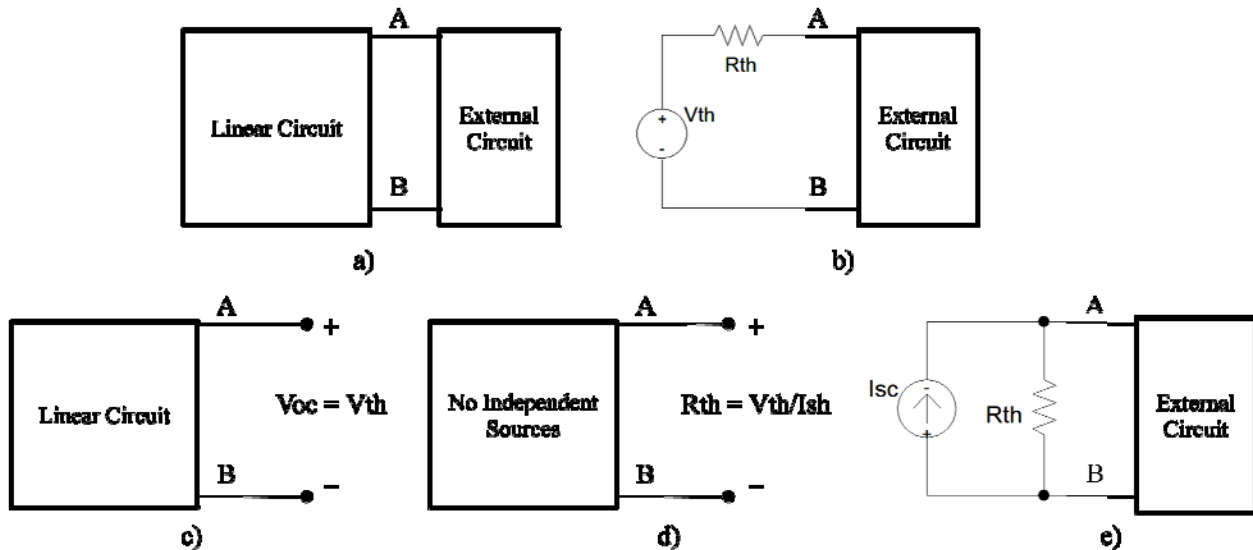


Figure 1 – Thévenin and Norton equivalent circuits.

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The Thévenin voltage V_{Th} is the open-circuit voltage across the two node terminals of the original circuit, as shown in Fig. 1c. The Thévenin resistance R_{Th} is the resistance “looking” into the two terminals A and B , as shown in Fig. 1d, with all independent sources deactivated: all the voltage sources are replaced by short circuits, and current sources by open circuits.

For Norton’s theorem, the equivalent circuit is a current source in parallel with a resistor of the same Thévenin resistance R_{Th} , as shown in Fig. 1e. The current I_{SC} of the current source is the current in a short circuit placed across terminals A and B , with a direction of A to B .

Maximum Power Transfer

In some circuits, maximum power transfer to a load resistor is desired. One example of such a circuit is in an audio system in which the load is the resistance of a speaker. It is desirable for the speaker to absorb the maximum power that the audio circuit can deliver.

As shown in Fig. 2, the circuit delivering power to the load can, for analysis purposes, be replaced by its Thévenin equivalent. Then, as proved in almost any circuits textbook, the load resistor that will absorb maximum power has a resistance equal to the Thévenin resistance. Specifically, for maximum power transfer,

$$R_L = R_{Th} ,$$

as shown in Fig. 2.

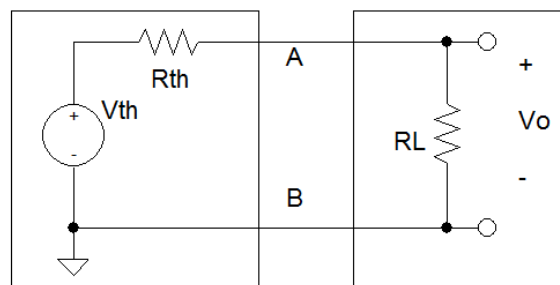


Figure 2 – Maximum power transfer occurs when $R_L = R_{Th}$

Combining the load resistance with the Thévenin resistance, the load current is

$$I = V_{TH} / (R_{TH} + R_{TH}) = V_{TH} / 2R_{TH} .$$

Thus, the power absorbed by the load resistor is

$$P = I^2 R_L = [V_{TH} / (R_{TH} + R_L)]^2 R_L .$$

The maximum power transferred to a matching load ($R_L = R_{TH}$) is therefore given by

$$P_{MAX} = V_{TH}^2 / 4R_{TH} .$$

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Wheatstone Bridge and Resistance Measurements

In 1843, the English scientist Charles Wheatstone invented the bridge circuit bearing his name. It is widely used for rapid and precise resistance measurements. Other similar bridges can be used to measure capacitance or inductance.

Although a commercial Wheatstone bridge may contain precision resistors, switches, and potentiometers to protect the meter movement, and perhaps a variable voltage source, the simplified circuit of Fig. 3 will be used here to illustrate the basic principles. The resistances R_n , R_m , and R_p are known, and the resistance R_x is to be determined. The resistance R_p is variable. The meter between nodes b and c can be a sensitive ammeter or voltmeter.

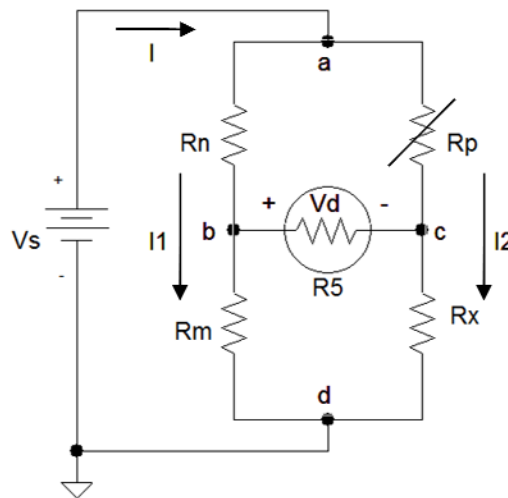


Figure 3 – Simplified Wheatstone bridge circuit for measuring resistance.

In the use of the bridge, the resistance of R_p is adjusted until the meter movement shows no deflection. Then V_d , the detector voltage, is zero, which means that nodes b and c must be at the same potential. Consequently, the voltage drop from a to b equals that from a to c . Also, the voltage drop from b to d equals that from c to d . Further, since the meter current is zero, the current I_1 flows through resistor R_n and also through resistor R_m . Similarly, the current I_2 flows through resistors R_p and R_x . Since $V_{ab} = V_{ac}$, then $I_1 R_n = I_2 R_p$. Also, since $V_{bd} = V_{cd}$, then $I_1 R_m = I_2 R_x$. The division of equations yields:

$$I_2 R_x / I_2 R_p = I_1 R_m / I_1 R_n ,$$

which reduces to

$$R_x = R_p (R_m / R_n) \text{ (bridge balance equation).}$$

This is the *bridge balance equation*. With the bridge balanced, this equation can be used to determine the resistance R_x to a precision that is nominally the same as the precision of the resistances R_m , R_n , and R_p .

During the preliminary adjustments, when the bridge may be far from a balanced condition, the meter movement must be protected by a variable series resistor the resistance of which is large compared with that of the meter. If, however, the voltage supply V_s is variable, the variable series resistor may not be

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needed. Then, the voltage V_s can be gradually increased from zero to some maximum voltage, with repetitive attempts to balance the bridge at intermediate values of voltage. This is an equally safe and accurate method for balancing the bridge.

PRE-LAB AND QUESTIONS

1. Find the voltage V_o across 2.2-k Ω resistor in the circuit shown in Fig. 4 analytically.
2. Use Multisim to construct the circuit of Fig. 4.
 - a. Measure V_o in your simulation and take a screenshot of this measurement.
 - b. Replace the 9V source with a short-circuit and measure the portion of V_o contributed by the 15V source. Take a screenshot of this measurement.
 - c. Replace the 15V source with a short-circuit and measure the portion of V_o contributed by the 9V source. Take a screenshot of this measurement.
 - d. Find V_o in the original circuit by adding the voltages calculated in parts b and c. Compare this answer with the one you got in part a. You have just demonstrated superposition!
 - e. For the circuit in Fig. 4, replace R4 with an open circuit and measure the resulting voltage at the open circuit. Take a screenshot of this measurement.
 - f. For the circuit in Fig. 4, replace R4 with a short circuit and measure the resulting current in that short. (In the actual circuit, you could use your DMM as your short circuit.). Take a screenshot of this measurement.
 - g. Find the Thévenin's and Norton's equivalent circuits as seen by the 2.2 k Ω load resistance. Build the Thévenin's equivalent circuit in Multisim using the 2.2 k Ω resistor as your load.. (Hint: See the schematic in Fig. 1b.) Measure the current through the load resistor and take a screenshot of this value. How does this compare to the value you found in step f?
 - h. Plot the power absorbed by the load as a function of the load resistance R_L for V_{TH} and R_{TH} by varying R_L from 1 k Ω to 10 k Ω . Determine, from the graph, the maximum power and the corresponding value of R_L . For what resistance is the maximum power absorbed?
3. Repeat part 2 b-g analytically, i.e., with equations, and report any significant difference with what was found in part 2.

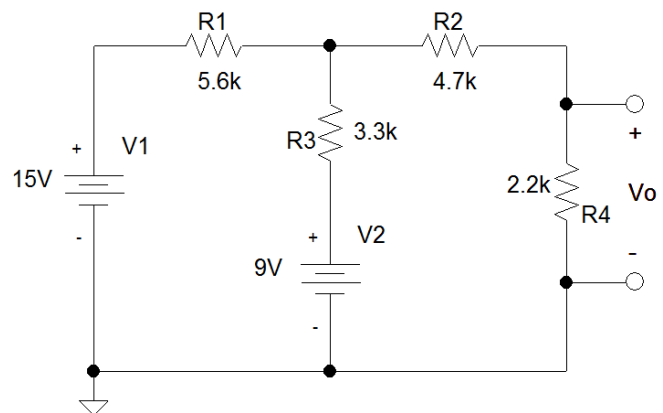


Figure 4 – Example circuit for pre-lab analysis.

LAB PROCEDURE AND QUESTIONS

The Superposition Theorem

1. Build the circuit of Fig. 4 and carry out steps 1b, 1c, and 1d of the pre-lab assignment by measuring the individual voltage contributions.

CAUTION: Replace the voltage sources with short circuits. Do not place a short circuit across a voltage source because the resulting large current will destroy the voltage source.

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- Determine the voltage V_o across the 2.2-k Ω resistor by algebraically adding these two measured voltages. Compare the result with the calculated value and also with the measured value of V_o with both sources in the circuit.

Thévenin's and Norton's Theorems

- Construct the circuit of Fig. 4. The 2.2-k Ω resistor will be considered to be the load.
- Using the DMM (digital multimeter) ammeter, measure the current flowing through the 2.2-k Ω resistor.
- Remove the 2.2-k Ω resistor and use the DMM voltmeter to measure the Thévenin voltage.
- Remove the 2.2-k Ω resistor and use the DMM ammeter to measure the short-circuit current I_{SC} . Find R_{Th} again, but do so by using $R_{Th} = V_{Th}/I_{SC}$. Does this value agree with the calculated value?
- Adjust the power supply to V_{Th} and a potentiometer or decade resistance box to R_{Th} , and connect them in series with the 2.2-k Ω resistor. Using the DMM ammeter, measure the current through the 2.2-k Ω resistor. Does this value agree with the value from step 2?

Maximum Power Transfer

- Build the Thévenin equivalent of the circuit of Fig. 4.
- Using a number of load resistances, with one of the values near R_{Th} , find the power absorbed by each resistor by using two out of three measurements of load voltage, load current, and load resistance. Have three values of resistances greater than R_{Th} , and three less. Make sure to spread out resistance values.
- Using the values obtained in Step 2, make a plot of power absorbed versus load resistance. Does the plot peak at R_{Th} ? Is the plot symmetrical about the peak value? Do you expect it to be? Explain.

Wheatstone Bridge and Resistance Measurements

Construct a Wheatstone bridge circuit as shown in Fig. 3. Use the digital multimeter (DMM) for measuring dc current and voltage. Use a variable dc power supply for V_s . Use standard carbon resistors for resistors R_m and R_n , and accurately measure their resistances with an ohmmeter (DMM) before assembling the bridge. The individual resistance values of R_m and R_n are not critical, but, as can be shown by a network sensitivity analysis, the ratio R_m/R_n should be between 0.4 and 6.0 for good results. Use a precision decade resistor for R_p . Of course, the resistor R_x is the resistor of initially unknown resistance.

In using the bridge to determine R_x , start out with V_s at zero, and gradually increase it while simultaneously balancing the bridge by adjusting R_p . Make certain that the meter (DMM) is connected across the bridge (b to c). Then, adjust the variable resistor R_p to obtain zero volts. When balance is obtained for a sizable V_s , determine the resistance R_x from the bridge balance equation.

Specifically,

- Use the bridge to measure the resistances of at least two miscellaneous resistors. Then, measure the resistances with a DMM and compare.
- Determine the percentage variation of the measured value from the nominal resistance of each resistor. Comment on any variations of the results.