Lecture #9(b): Discrete-Time Signals

1. Introduction

Now that we have explored the process of sampling — that is, the conversion of continuous-time signals to discrete-time signals,

$$x[n] = x_c(n/f_s) \tag{1}$$

— we are ready to cover some important aspects of discrete-time signals, as we did with continuous-time signals. Specifically, this set of notes covers the following topics:

1. Mathematical representation and transformations of discrete-time signals.
2. Some important discrete-time functions.

This introductory treatment closely parallels our previous treatment of continuous-time signals.

2. Discrete-time signals

A. Signal transformations

Let $x[n]$ denote a discrete-time function of time index n. As we have already seen for continuous-time functions, it will frequently be important to know how the function x changes when we change its argument. The table below gives the qualitative effect of some simple changes in argument for discrete-time signals.

discrete-time function	*effect*
$x[-n]$	Reflection
$x[an]$, $a \in \{2, 3, \ldots\}$	Compression
$x[n-a]$, $a \in \{1, 2, 3, \ldots\}$	Shift to the right along the horizontal axis
$x[n+a]$, $a \in \{1, 2, 3, \ldots\}$	Shift to the left along the horizontal axis
$a \cdot x[n]$, $a > 1$	Magnification
$a \cdot x[n]$, $a < 1$	Reduction
$x[n] + a$, $a > 0$	Shift up along the vertical axis
$x[n] - a$, $a > 0$	Shift down along the vertical axis

Figures 1 and 2 illustrate some of these on two simple discrete-time functions, which are sampled versions of the continuous-time functions in Figures 1 and 2 of the “Lecture #6” notes, with sampling frequencies of 10Hz and 5Hz, respectively. It is very important that you understand each of these illustrations, and are able to perform them yourself without the aid of a computer or calculator.

As was the case for continuous-time signals, compound transformations that perform both scaling and left/right shifting are a little trickier than each one by itself. Consider $x[n]$ in Figure 1 and the compound transformation $x[2n-10]$. To understand what this function looks like, we first change it to:

$$x[2n-10] = x[2(n-5)] \tag{2}$$

In this form, we see that we first scale the function, and then shift the scaled function by 5 units (*not* 10 units) to the right; this transformation is illustrated in the bottom right corner of Figure 1. Figure 2 (bottom right corner) illustrates another compound-transformation example:

$$x[-n+20] = x[-(n-20)] \tag{3}$$

Again, we see that by factoring the scaling information (in this case a reflection), the function is first reflected about the y-axis, and is then shifted 20 time units to the right (*not* to the left). (The *Mathematica* notebook “discrete_transformations.nb was used to generate these examples.)

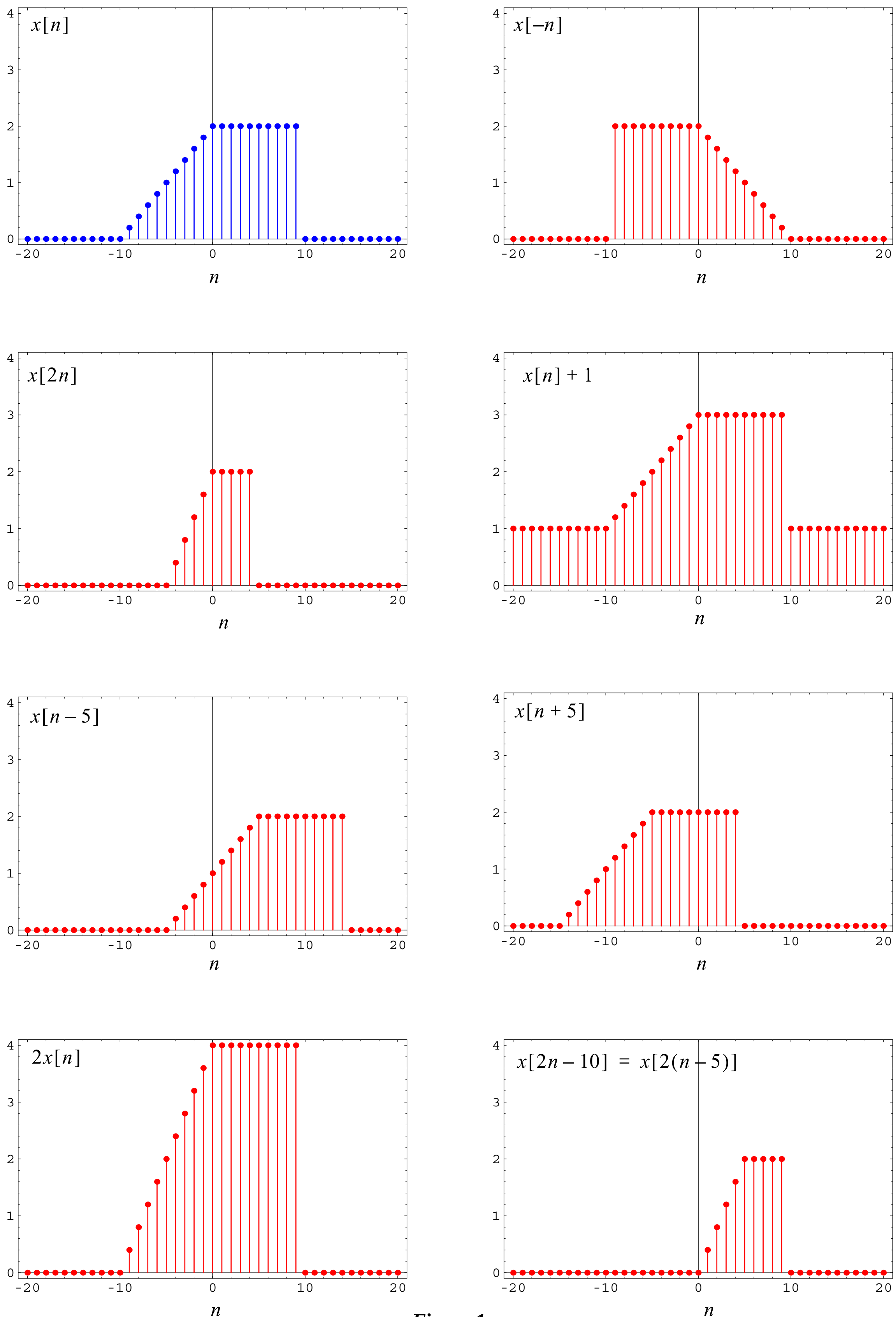

Figure 1

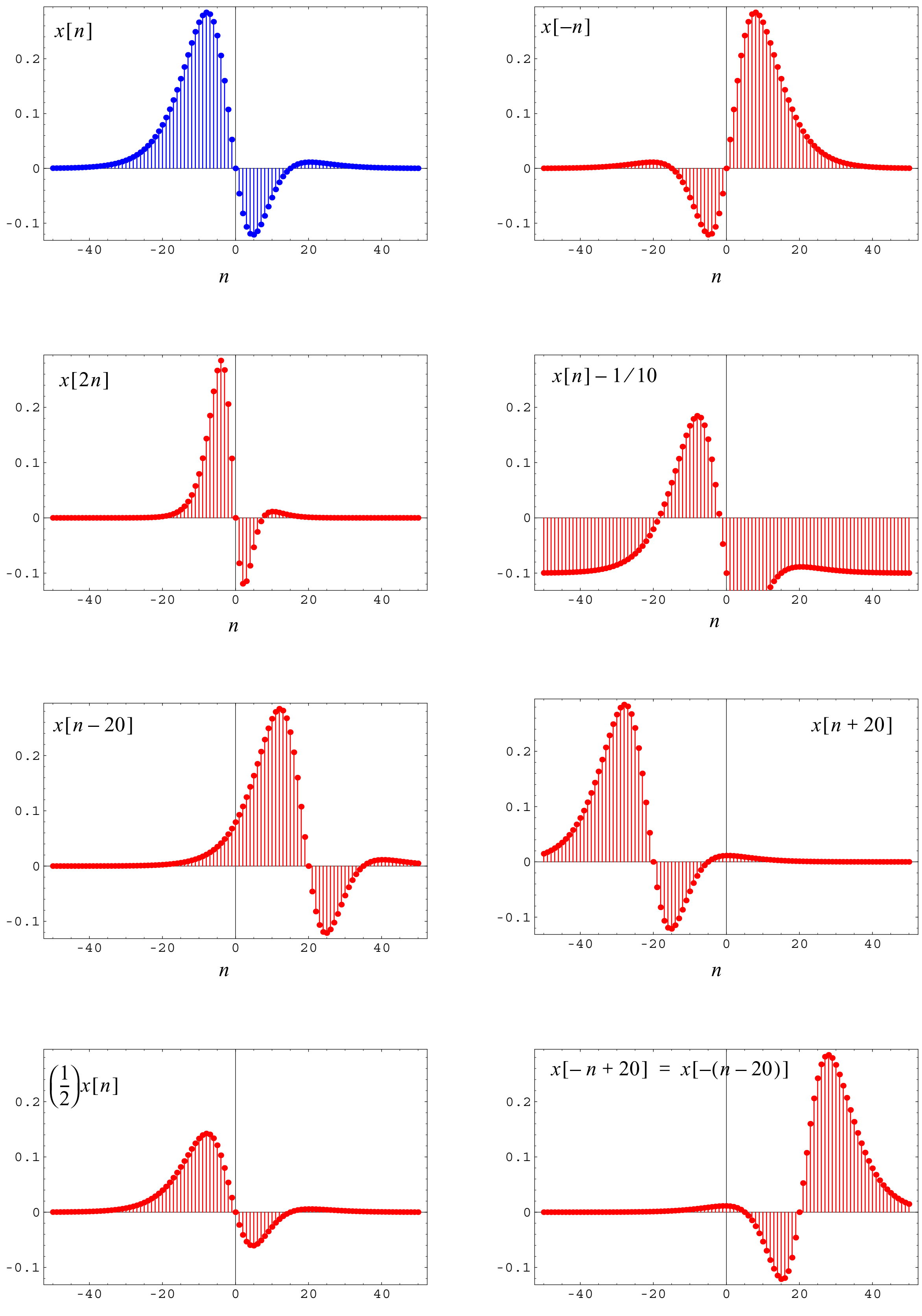

Figure 2

B. Some useful discrete-time signals

In this section, we introduce some very useful discrete-time signals. The first of these is the *discrete-time impulse* or *delta* function $\delta[n]$, defined by,

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}, \tag{4}$$

and plotted in Figure 3 below.

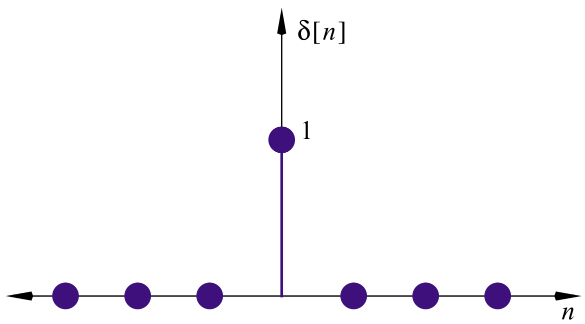

Figure 3

We can define any discrete-time signal as the weighted sum of time-shifted δ functions:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{5}$$

For example, the discrete-time signal $x[n]$ in Figure 4 below can be written as:

$$x[n] = \delta[n+1] + 2\delta[n] + 2\delta[n-1] - \delta[n-2] \tag{6}$$

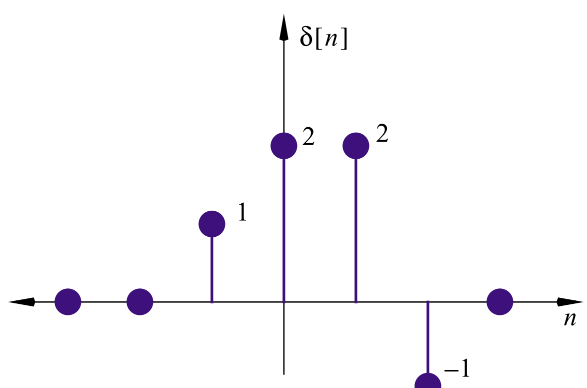

Figure 4

Another discrete-time function of significance in our mathematical representation of signals is the *discrete unit step* function $u[n]$,

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{7}$$

plotted in Figure 5.

Finally, we introduce the discrete-time sinusoidal function. Since the continuous-time sinusoid can be written as,

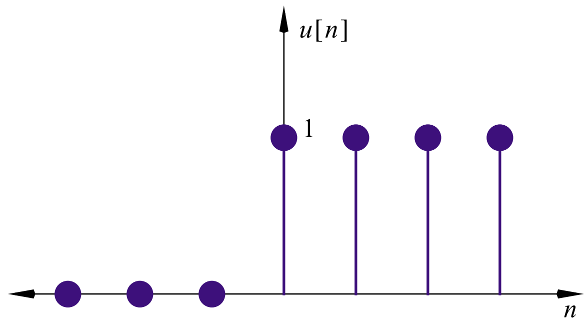

Figure 5

$$x_c(t) = A\cos(2\pi f_o t + \alpha) \tag{8}$$

the discrete-time equivalent is given by,

$$x[n] = x_c(n/f_s) = A\cos(2\pi f_o(n/f_s) + \alpha) \tag{9}$$

In Figure 6 below, we plot equation (8) for $f_o = 1\text{Hz}$ and different sampling frequencies f_s.

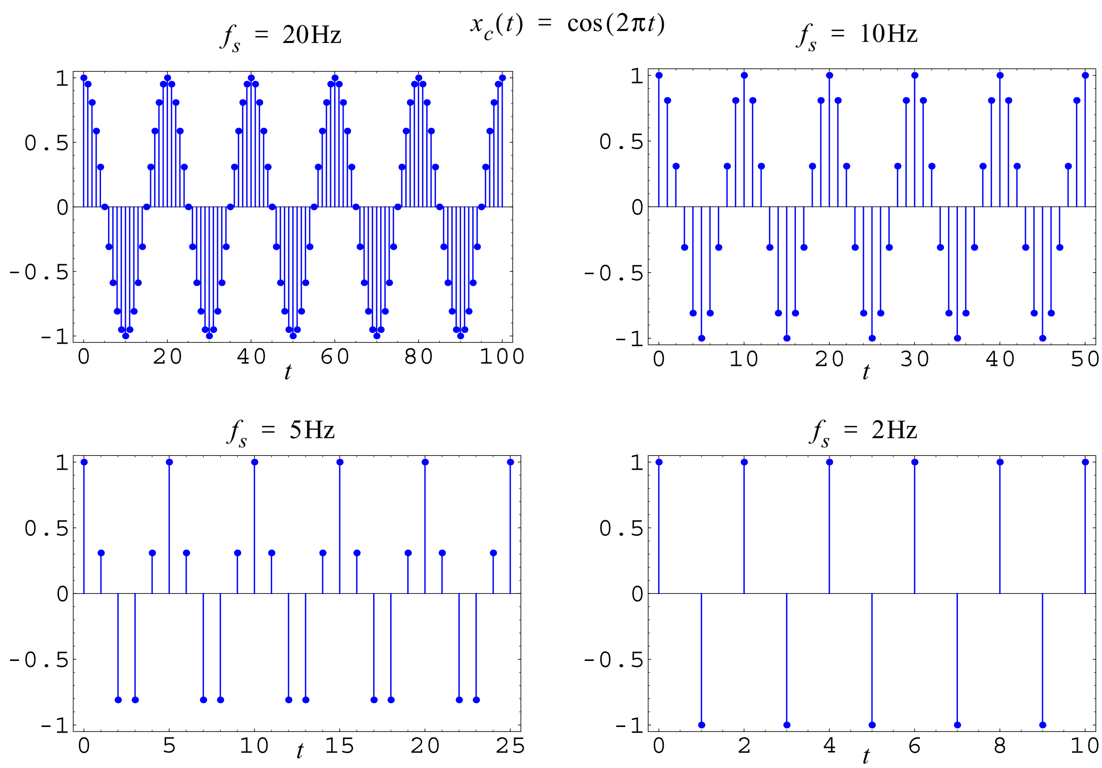

Figure 6