EEL6825: Pattern Recognition

Quiz I

Fall 2003

Problem 1. (5 points)

When rolling an unbiased die 6 times, which of the following outcomes has a higher probability? Why?

1. The outcome has three 1's and three 6's.
2. The outcome is the sequence $\{1, 2, 3, 4, 5, 6\}$.

Problem 2. (25 points)

Let two random variables, $\mathbf{x}$ and $\mathbf{y}$, have a joint probability distribution function (pdf)

$$p_{\mathbf{xy}}(x, y) = \frac{1}{8\pi} e^{-\frac{x^2+y^2+6x-2y+10}{8}} \tag{1}$$

Answer the following questions:

1. What type of distribution is $p_{\mathbf{xy}}(x, y)$? (5 points)
2. Are $\mathbf{x}$ and $\mathbf{y}$ independent? (5 points)
3. Are $\mathbf{x}$ and $\mathbf{y}$ correlated? (5 points)
4. What are the mean and variance values of $\mathbf{x}$? (10 points)

Problem 3. (35 points)

The two classes, ω_1 and ω_2, of the random variable $\mathbf{x}$ are distributed, as shown in Figure 1. The prior probabilities of the two classes ω_1 and ω_2 are equal.

1. Compute the constant k. (5 points)
2. Derive the Bayes classifier to classify data $\mathbf{x}$ into the two classes ω_1 and ω_2. (10 points)
3. Sketch a graph that indicates the Bayes error for the classifier from 2. Mark all the important points in the graph. (10 points)
4. Compute the Bayes error for the classifier from 2. (10 points)

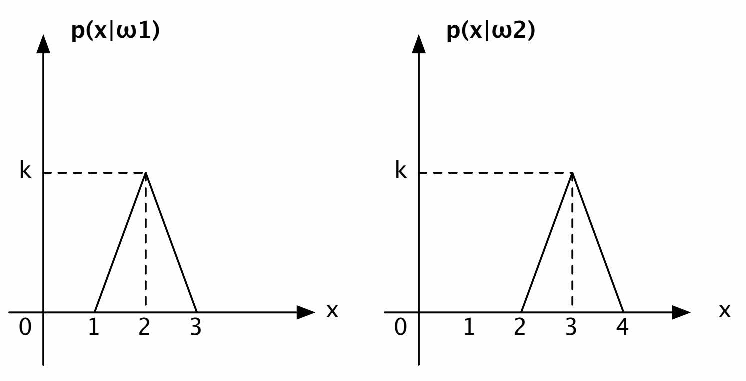

Figure 1: The two distributions for Problem 3.

Problem 4. (35 points)

Let a 2-D random vector $\mathbf{U}$, defined as $\mathbf{U} = [\mathbf{x}\ \mathbf{y}]^T$, have a uniform distribution:

$$p_{\mathbf{U}}(U|\theta_x, \theta_y) = \begin{cases} k & , \quad \theta_x - 0.5 \leq \mathbf{x} \leq \theta_x + 0.5\ ,\ \theta_y - 1.5 \leq \mathbf{y} \leq \theta_y + 1.5\ , \\ 0 & , \quad otherwise\ , \end{cases} \tag{2}$$

where k denotes a constant, and θ_x and θ_y are parameters of the distribution $p_{\mathbf{U}}(U|\theta_x, \theta_y)$. Further, suppose that the following 5 samples of $\mathbf{U}$:

$$U_1 = \begin{bmatrix} 0.23 \\ 2.79 \end{bmatrix}\ ,\ U_2 = \begin{bmatrix} -0.31 \\ 1.68 \end{bmatrix}\ ,\ U_3 = \begin{bmatrix} 0.04 \\ 3.25 \end{bmatrix}\ ,\ U_4 = \begin{bmatrix} -0.62 \\ 0.99 \end{bmatrix}\ ,\ U_5 = \begin{bmatrix} 0.17 \\ 3.72 \end{bmatrix}\ ,$$

are drawn independently from the distribution $p_{\mathbf{U}}(U|\theta_x, \theta_y)$.

1. Find the constant k (5 points)
2. Compute the maximum likelihood estimate (ML) of the parameters θ_x and θ_y, using the given independent samples $U_i,\ i \in [1, 5]$. (30 points)