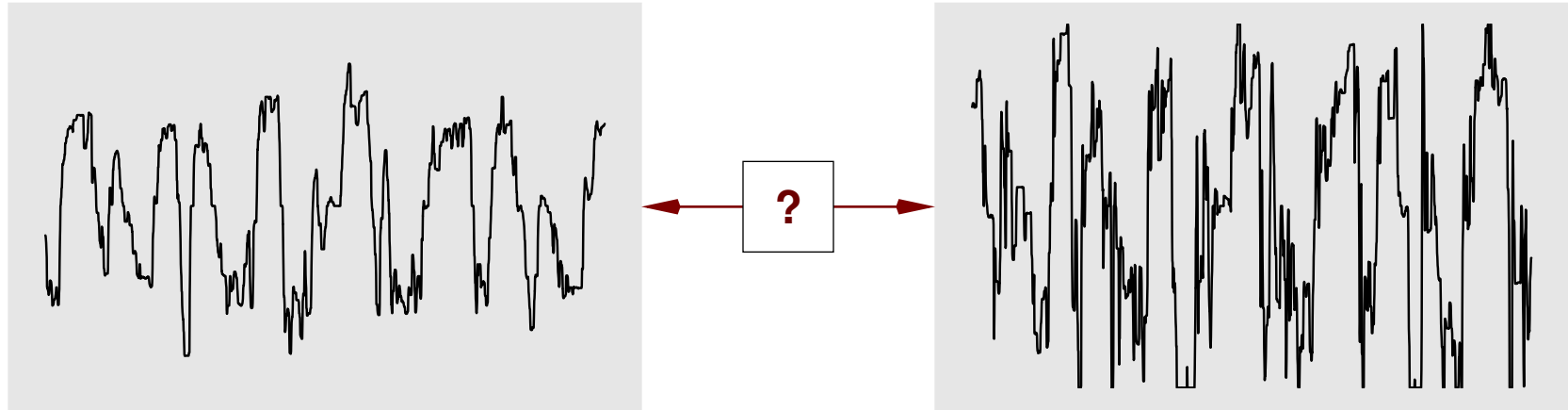
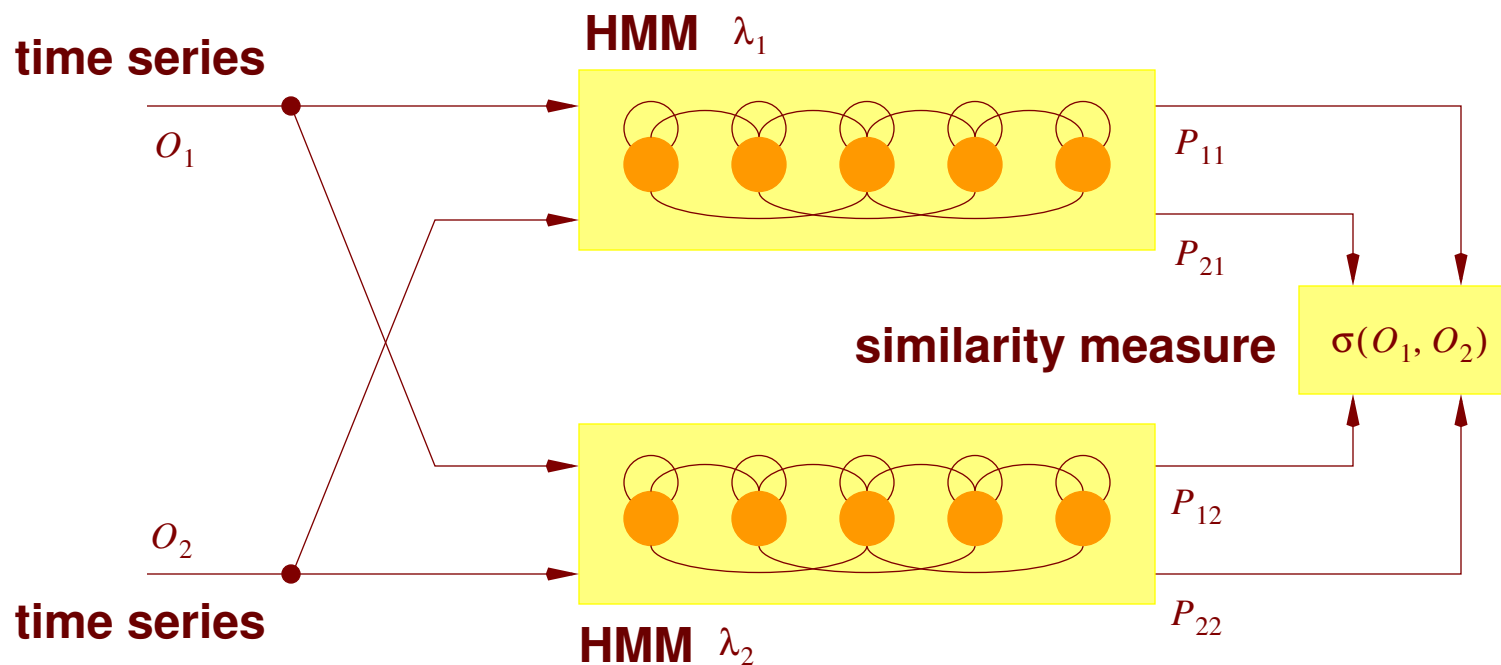


Similarity between two time series



HMM-based similarity measure



Definitions:

$$P(O_i|\lambda_i) > P(O_i|\lambda), \lambda \neq \lambda_i$$

$$P_{ij} = P(O_i|\lambda_j)^{1/T_i}$$

$$\sigma(O_1, O_2) = \sqrt{\frac{P_{21}P_{12}}{P_{11}P_{22}}}$$

Properties

Givens:

$$P_{ii} \geq \frac{1}{L} \text{ (global maximum assumption)}$$

$$P_{ii} \geq P_{ij} > 0 \text{ (smoothed HMMs)}$$

Similarity measure properties:

$$(1) \sigma(O_1, O_2) = \sigma(O_2, O_1)$$

$$(2) 0 < \sigma(O_1, O_2) \leq 1$$

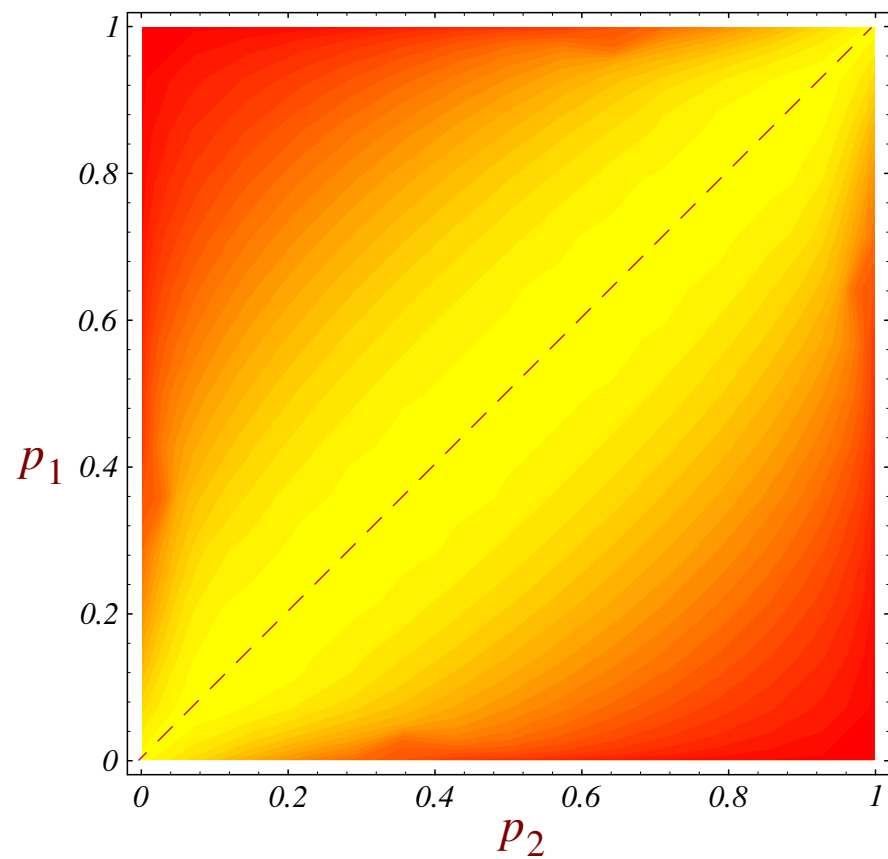
$$(3) \sigma(O_1, O_2) = 1 \text{ if and only if } \lambda_1 \sim \lambda_2$$

Variation with distribution

Single state HMMs with two different distributions

$$B_1 = \begin{bmatrix} p_1 \\ 1 - p_1 \end{bmatrix}$$

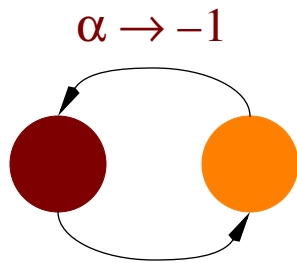
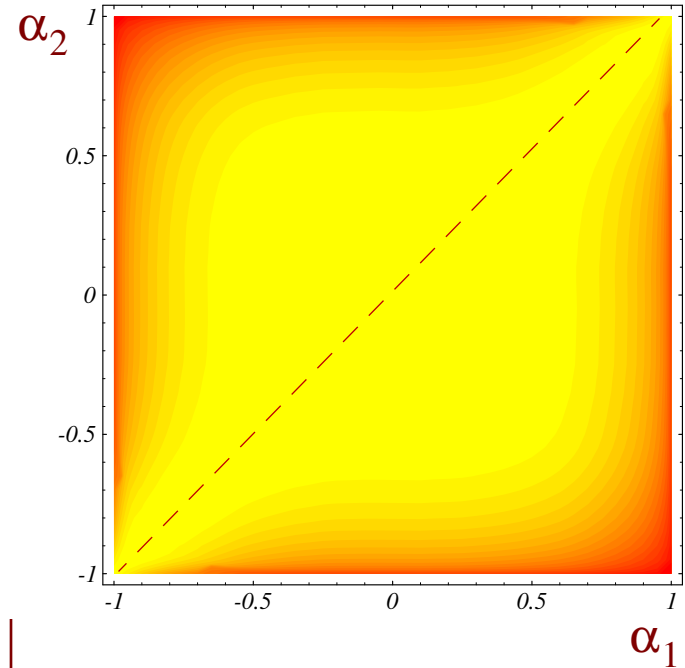
$$B_2 = \begin{bmatrix} p_2 \\ 1 - p_2 \end{bmatrix}$$



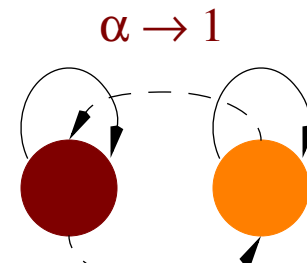
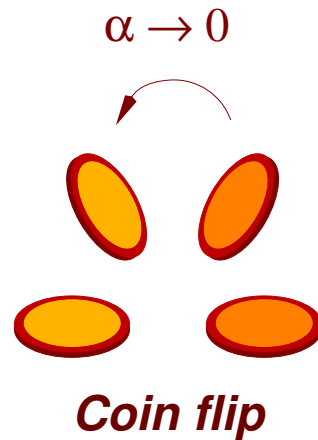
Variation with structure

Two-state HMM

$$\lambda(\alpha) = \left\{ \begin{matrix} \begin{bmatrix} \frac{1+\alpha}{2} & \frac{1-\alpha}{2} \\ \frac{1-\alpha}{2} & \frac{1+\alpha}{2} \end{bmatrix}, & \begin{bmatrix} \frac{1+\alpha}{2} & \frac{1-\alpha}{2} \\ \frac{1-\alpha}{2} & \frac{1+\alpha}{2} \end{bmatrix}, & \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \end{matrix} \right\}$$

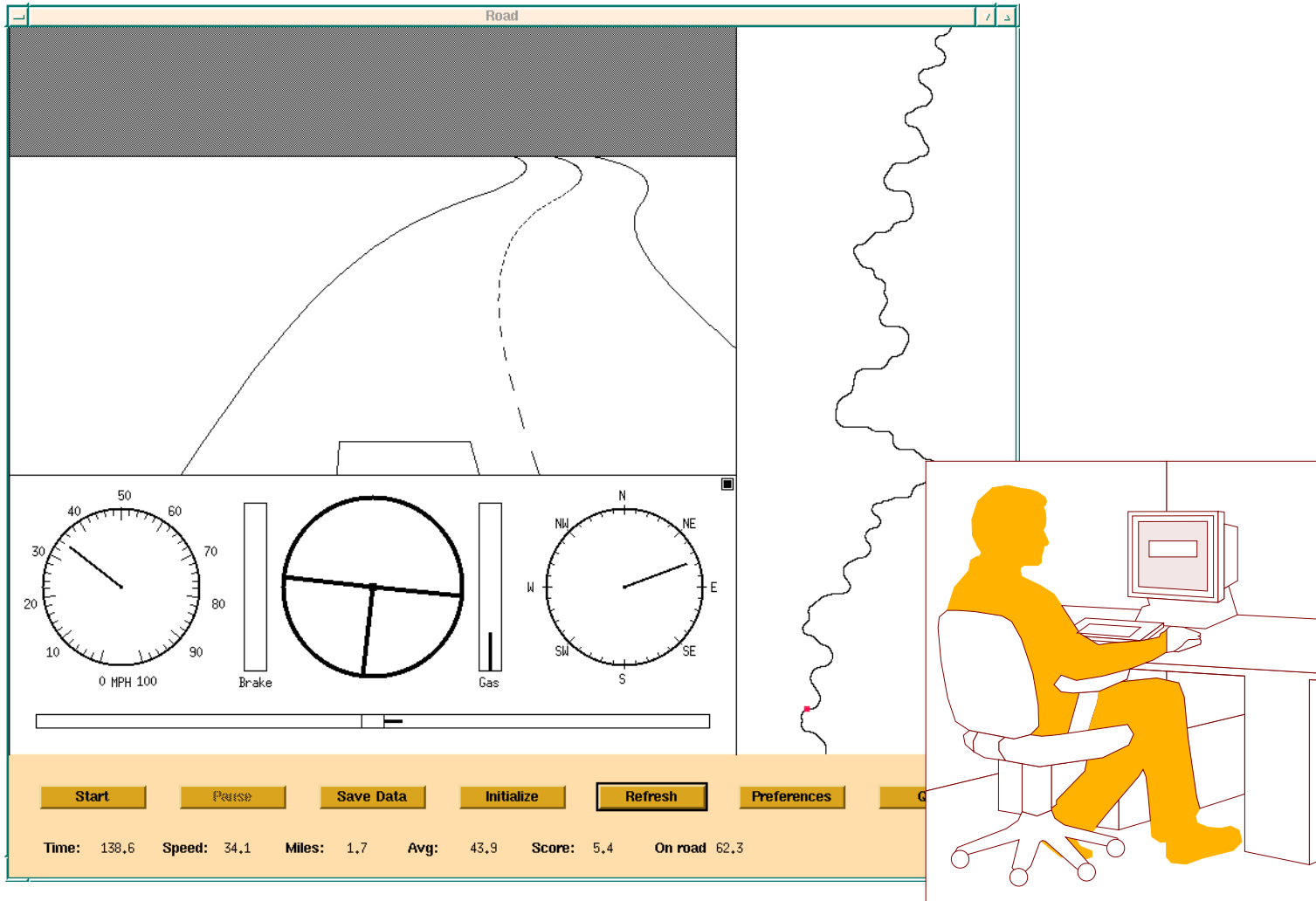


$\{ \dots, 1, 0, 1, 0, 1, 0, 1, 0, \dots \}$

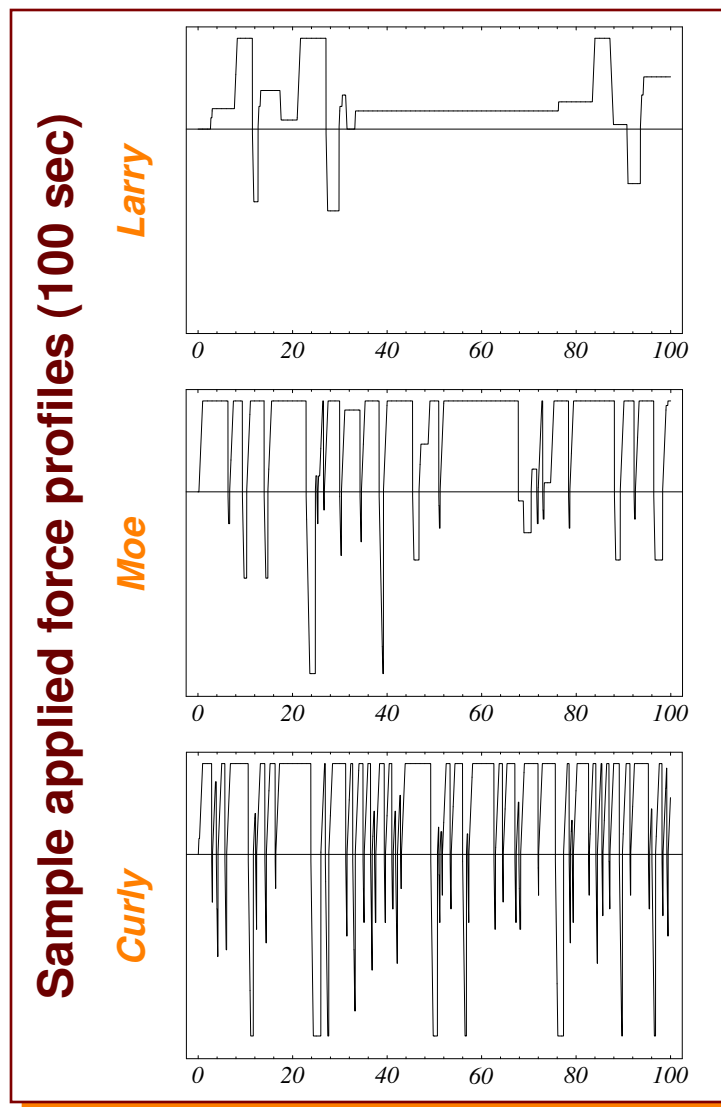
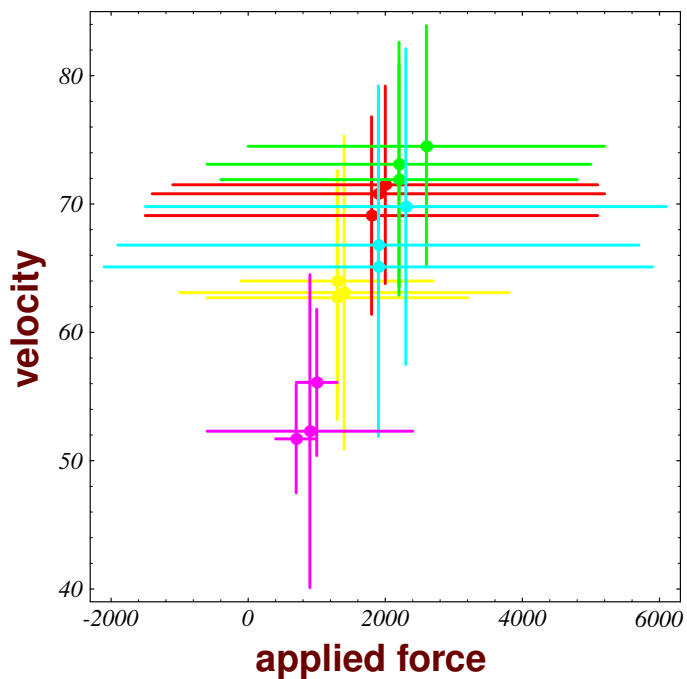


$\{ \dots, 1, 1, 0, 0, \dots, 0, 0, 1, 1, \dots \}$

Similarity measure for human driving data



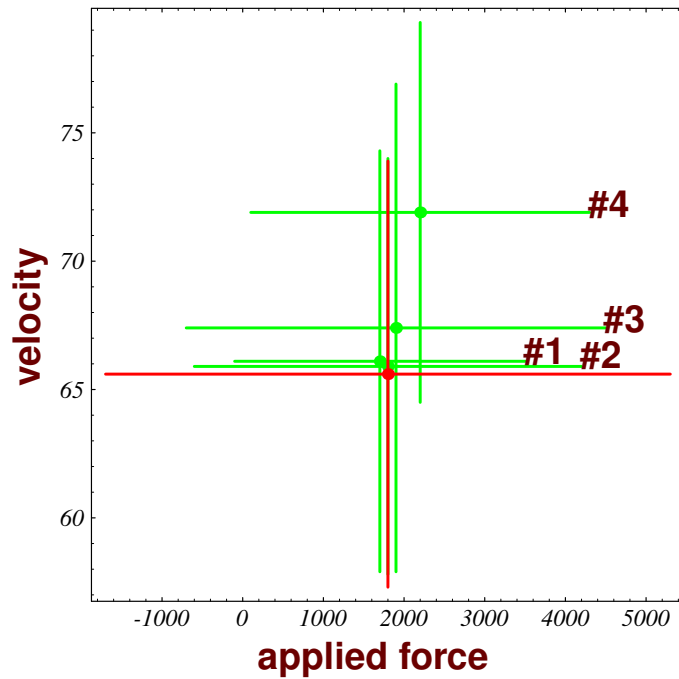
Human driving data



Classification performance

<i>Error rates</i>	<i>Similarity measure</i>	<i>Gaussian classifier</i>
<i>Sets #1, #2, #3</i>	<i>0/30 (0%)</i>	<i>7/30 (23%)</i>
<i>Turn maneuvers</i>	<i>0/10 (0%)</i>	<i>3/10 (30%)</i>
<i>Difficult classes</i>	<i>1/6 (17%)</i>	<i>5/6 (83%)</i>

More difficult classifications



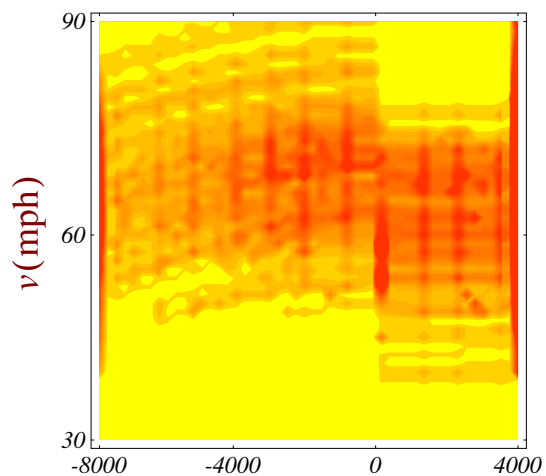
Similarity measure

	<i>Moe #4</i>	<i>Curly</i>
<i>Moe #1</i>	<i>0.616</i>	<i>0.315</i>
<i>Moe #2</i>	<i>0.603</i>	<i>0.495</i>
<i>Moe #3</i>	<i>0.760</i>	<i>0.550</i>

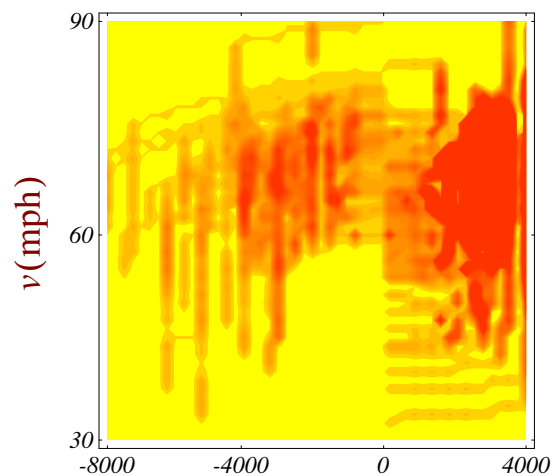
Gaussian classifier

	<i>Moe #4</i>	<i>Curly</i>
<i>Moe #1</i>	<i>0.431</i>	<i>0.569</i>
<i>Moe #2</i>	<i>0.337</i>	<i>0.663</i>
<i>Moe #3</i>	<i>0.433</i>	<i>0.567</i>

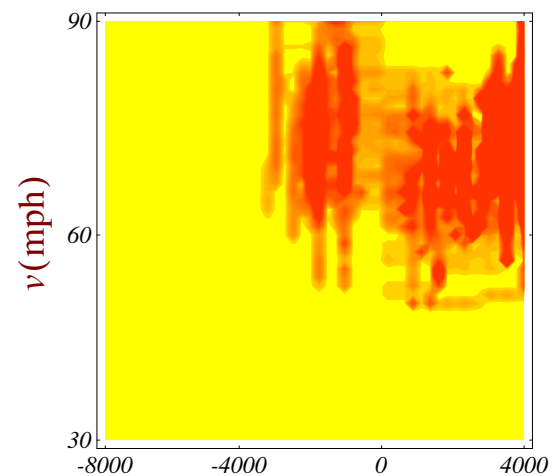
Similarity measure vs. Gaussian classifier



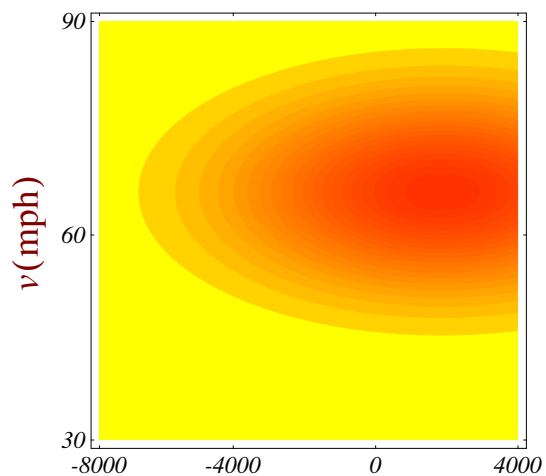
Curly's run



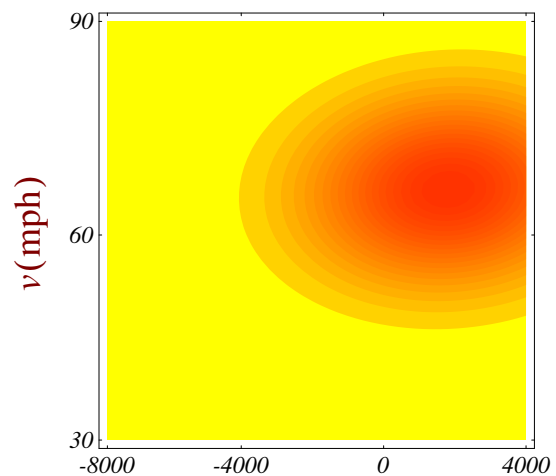
Moe's 2nd run



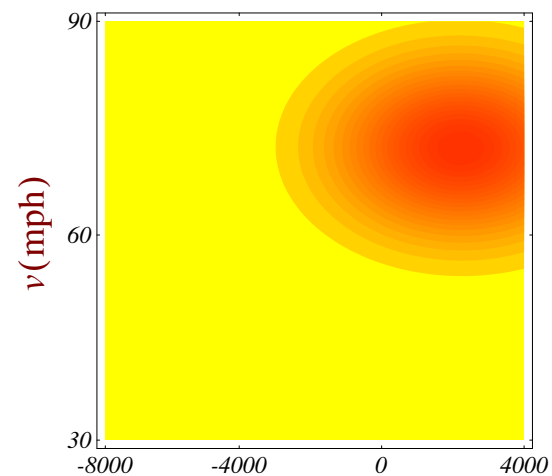
Moe's 4th run



applied force



applied force



applied force