EEL6562: Final Assignment (Spring 2004)

(distributed 04/20/2004, due 5:00PM, 05/01/2004)

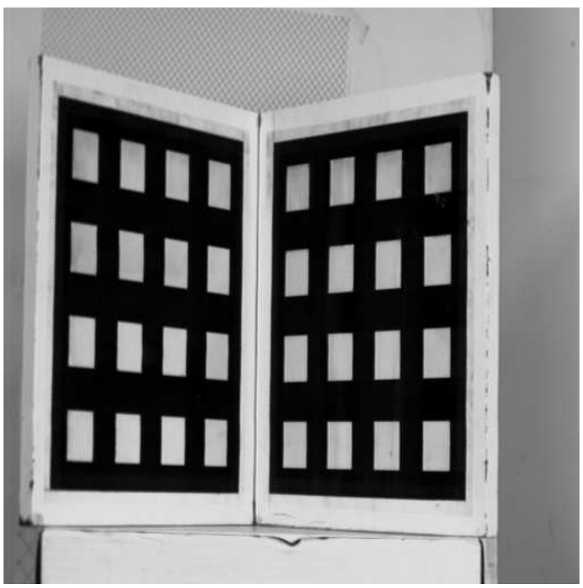
Note: This is an *individual* assignment, so please do your own work; starred (*) items are extra credit; about *half* of the points on this assignment are extra credit

1 Introduction

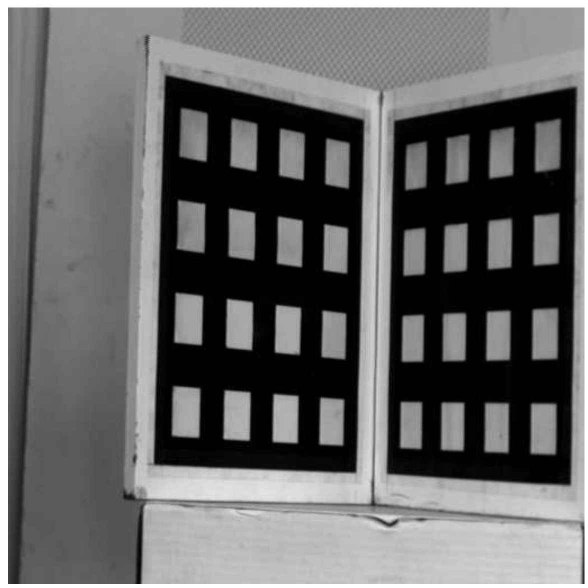
Goal: To conduct experiments with 3D computer vision algorithms.

For this assignment, you are given two 512×512 gray-scale images of an object, `view1.ppm` and `view2.ppm`, as shown in Figure 1. You are also given information about key *feature points* – namely, the 2D image coordinates and the 3D world coordinates of the corners of the white rectangles on the object [2]; both the images and these coordinate files are available on the course web site at:

`http://mil.ufl.edu/~nechyba/eel6562/assignments.html`



(a)



(b)

Figure 1: (a) view 1; (b) view 2.

The 2D image coordinate files, `2d_points1.txt` and `2d_points2.txt`, consist of 128 labeled image coordinates, corresponding to the corners of the white rectangles on the object for images `view1.ppm` and `view2.ppm`, respectively. Each line in these files has the following format:

```
label  x_image_coord  y_image_coord
```

The 3D world coordinate file `3d_points.txt` consists of 128 labeled 3D world coordinates corresponding to the 2D image coordinates. Each line in this file has the following format:

```
label  x_world_coord  y_world_coord  z_world_coord
```

Figure 2 illustrates the labels for each of the 128 feature points.

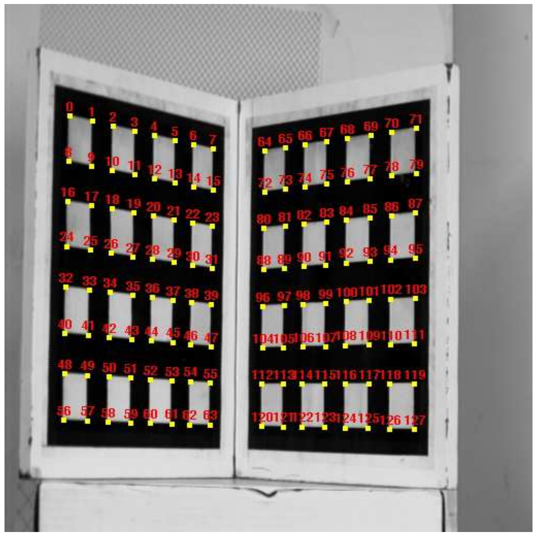


Figure 2: Labels for feature points.

2 Notation

Throughout this document, the following notation is used. The kth 2D image coordinate is denoted by (x'_k, y'_k) for `view1.ppm` (Figure 1a, `2d_points1.txt`) and (x_k, y_k) for `view2.ppm` (Figure 1b, `2d_points2.txt`). The homogeneous representation for the kth 2D image coordinate is denoted by $\mathbf{x}'_k$ for `view1.ppm` and $\mathbf{x}_k$ for `view2.ppm`, where,

$$\mathbf{x}'_k = \begin{bmatrix} x'_k \\ y'_k \\ 1 \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ 1 \end{bmatrix}. \tag{1}$$

The kth 3D world coordinate in `3d_points.txt` is denoted by (X_k, Y_k, Z_K) and its homogeneous representation is denoted by $\mathbf{X}_k$, where,

$$\mathbf{X}_k = \begin{bmatrix} X_k \\ Y_k \\ Z_k \\ 1 \end{bmatrix}. \tag{2}$$

The projective matrix P is a 3×4 matrix,

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \tag{3}$$

that defines the relationship between $\mathbf{x}_k$ and $\mathbf{X}_k$:

$$s\mathbf{x}_k = P\mathbf{X}_k \tag{4}$$

Similarly, for $\mathbf{x}'_k$ and $\mathbf{X}'_k$:

$$s\mathbf{x}'_k = P'\mathbf{X}'_k \tag{5}$$

Finally, the fundamental matrix F is a 3×3 matrix:

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \tag{6}$$

that defines the epipolar geometry between the two views:

$$\mathbf{x}'^T_k F \mathbf{x}_k = 0 \tag{7}$$

3 Problems

3.1 Part I: Camera calibration and 3D estimation

1. For the following estimations of P and P', use all 128 points.

   (a) (20 points) Estimate the projection matrices P and P', and normalize your answer such that $p_{34} = 1$.

   (b) (20 points) Estimate the 3D coordinates of all 128 labeled feature points. Report the following 3D estimation error measure E for each image:

   $$E = \sum_{k=0}^{127} d(\mathbf{X}_k, \hat{\mathbf{X}}_k)^2 \tag{8}$$

   where $\hat{\mathbf{X}}_k$ denotes your 3D estimate of point k, and $d(\mathbf{v}_1, \mathbf{v}_2)$ denotes the Euclidean distance between vectors $\mathbf{v}_1$ and $\mathbf{v}_2$:

   $$d(\mathbf{v}_1, \mathbf{v}_2) = \sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^T(\mathbf{v}_1 - \mathbf{v}_2)} \tag{9}$$

   (c) (20 points) Estimate the surface area A of the rectangular black surfaces on the object. To do this, you will need to estimate the four 2D image coordinates of the corners for each surface and each image (you can do this by hand), and estimate the corresponding 3D coordinates, as in the previous step. Are your 3D estimates of the corners co-planar?

2. *(15 points) Estimate the surface area A of the *visible* black areas of the black surfaces on the object.

3. *(25 points) Repeat parts 1a and 1b, except now use only the following specified subset of points in your estimation of the projection matrices P (however, still evaluate E over all 128 points):

   (a) Points with labels $k \in \{6, 7, 14, 15, 64, 65, 72, 73\}$.

   (b) Points with labels $k \in \{0, 5, 56, 61, 65, 71, 121, 127\}$.

   (c) Which subset of points above generates more error? Why?

   (d) Points with labels k, $k \in \{0, \ldots, 63\}$. Do you have a problem with this estimation? If so, why?

3.2 Part II: Fundamental matrix and epipolar geometry

1. (50 points) In the following experiments, you may make use of all available 2D information (i.e. 128 corresponding image points $(\mathbf{x}'_k, \mathbf{x}_k)$, $k \in \{0, \ldots, 127\}$).

   (a) Estimate the fundamental matrix F by applying the unnormalized eight-point algorithm. Scale your result such that $f_{33} = 1$.

   (b) For your result in part 1a, draw epipolar lines on both images (`view1.ppm` and `view2.ppm`) corresponding to image coordinates $k \in \{0, 23, 96, 127\}$.

   (c) Compute the cost function J,

   $$J = \sum_{k=0}^{127} d(\mathbf{x}'_k, F\mathbf{x}_k)^2 + d(\mathbf{x}_k, F^T\mathbf{x}_k)^2 \tag{10}$$

   where $d(\mathbf{x}, \ell)$ is defined as the distance between a homogeneous coordinate $\mathbf{x}$ and line ℓ (in homogeneous notation):

   $$d(\mathbf{x}, \ell) = \frac{\mathbf{x}^T \ell}{\sqrt{\lambda^2 + \mu^2}} \tag{11}$$

   for $\ell = \begin{bmatrix} \lambda & \mu & \nu \end{bmatrix}^T$.

2. *(10 points) Repeat part 1 using the normalized eight-point algorithm with either isotropic or non-isotropic normalization, as detailed in Section 5 of [1]. How does this result compare to part 1?

3. *(20 points) Repeat part 1 using the seven-point algorithm. How does this result compare to part 1?

4. *(20 points) Improve your estimate in part 1 using nonlinear minimization of J in equation (10).

4 Submission

You need to send an email to `nechyba@mil.ufl.edu` of a web link (URL) that points to either an `.html` or `.pdf` report of your work by the specified due date. In either case, your report should:

1. Include your full name;
2. Discuss your results and explanations for each of the problems;
3. Discuss algorithms/code that you implemented for this assignment; and
4. Provide links to any code that you implemented for this assignment.

References

[1] R. I. Hartley, "In Defense of the Eight-Point Algorithm," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 19, no. 6, pp. 580-93, 1997.

[2] Z. Zhang, `http://www-sop.inria.fr/robotvis/personnel/zzhang/CalibEnv/CalibEnv.html`, April 2004.