Lab 6: RC Transient Circuits

OBJECTIVES

- Understand RC transient circuits.
- Determine the time constant of a circuit through simulation, experiment, and analytically.
- Understand the differentiating and integrating RC circuits and how the performance is affected by frequency and the time constant.

MATERIALS

- Pre-lab questions and Multisim screenshots.
- Your lab parts.
- Printouts (required) of the below documents:
  - Pre-lab analyses
  - Answers to pre-lab questions
  - Multisim screenshots e-mailed to course e-mail
- Graph paper.

INTRODUCTION

A Simple RC Circuit

The capacitor has a wide range of applications in electronic circuits, some of which are energy storage, dc blocking, filtering, and timing. Thus, it is important for engineering students to understand capacitor operation. This experiment is designed to familiarize the student with the simple transient response of two-element RC circuits, and the various methods for measuring and displaying these responses.

Case 1: Capacitor is Charging

In normal operation, a capacitor *charges* part of the time and *discharges* at other times. Consider first the charging process. In the circuit of Fig. 1, for t < 0, both of the switches are open and no energy is stored on the capacitor. We say that the initial conditions are zero, or $v_o(0) = 0$. At time t = 0, switch S1 closes and the capacitor begins charging.

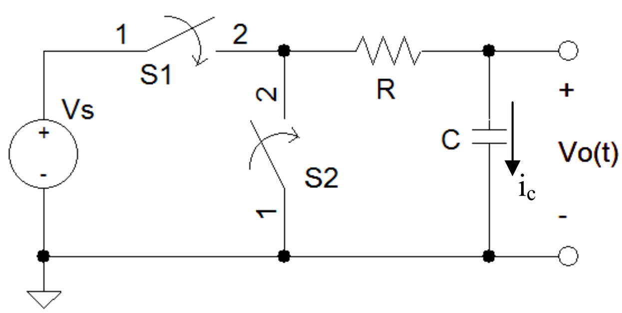

Figure 1 – Series *RC* circuit.

In deriving the circuit equation for this circuit, we will use the current/voltage relationship for a capacitor:

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

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That is, current through the capacitor is proportional to the time derivative of the voltage across the capacitor. The coefficient *C*, is the capacitance measured in farads. Applying *KCL* at the upper capacitor node (for $t > 0$) yields

$$i_c(t) + \frac{v_o(t) - V_S}{R} = 0$$

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_S}{R} = 0$$

$$\frac{dv_o(t)}{dt} + \frac{v_o(t)}{RC} = \frac{V_S}{RC}$$

Note that the capacitor voltage v_c, is the same as the output voltage v_o. The solution of this linear, constant-coefficient differential equation is

$$v_o(t) = V_S\left(1 - e^{-\frac{t}{RC}}\right) \qquad for\ t > 0$$

An important quantity for an RC circuit is known as the *time constant*, $\tau=RC$, so the above equation can be written as

$$v_o(t) = V_S\left(1 - e^{-\frac{t}{\tau}}\right) \qquad for\ t > 0$$

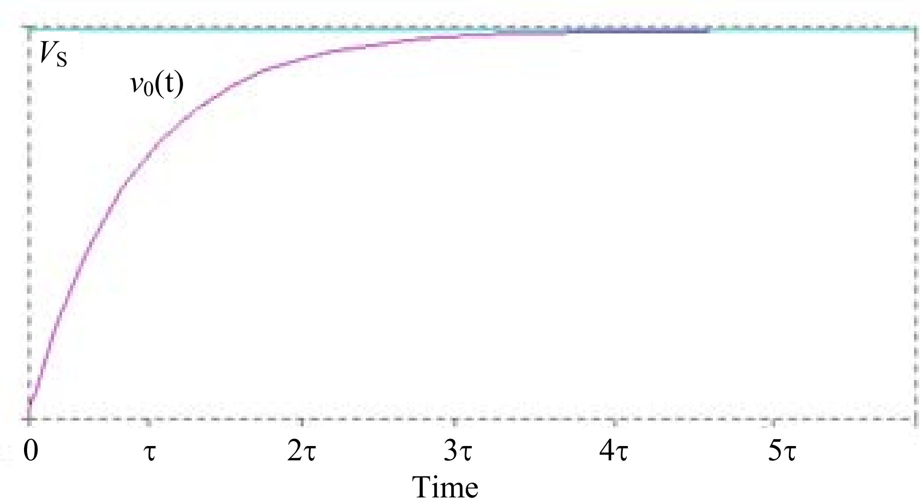

Figure 2 – Plot of capacitor output voltage with time for the charging case.

As seen in Fig. 2, the charging capacitor approaches the source voltage V_S, as $t \rightarrow \infty$. When can we say that the capacitor is "close enough" to the final value? The time constant provides a measure of the system's response to change. The time constant, is defined as the time at which the system has come to within 1/e of its final (asymptotic) value. In the case of our simple RC circuit, we can solve for τ by setting the output voltage to V_S minus 1/e times V_S

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$$v_o(\tau) = V_S - \frac{1}{e}V_S = V_S\left(1 - e^{-\frac{\tau}{RC}}\right)$$

$$e^{-1} = e^{-\frac{\tau}{RC}}$$

$$\tau = RC$$

For practical purposes, the circuit is considered fully charged (or discharged) after 5 time constants.

$$v_o(5\tau) = V_S\left(1 - e^{-\frac{5\tau}{RC}}\right)$$

$$= V_S\left(1 - e^{-\frac{5RC}{RC}}\right)$$

$$= V_S(1 - e^{-5})$$

$$= 0.993V_S$$

Therefore, it is agreed that we are "close enough" after 5 time constants, when the capacitor is 99.3% charged.

When the exponent of $e^{-t/\tau}$ equals, i.e., $-t/2 = -2$, two time constants have elapsed, and so on. At one time constant, an evaluation of e^{-1} shows that the charging curve has risen to 63.2% of the maximum amplitude (see Fig. 2). In a discharging circuit, at one time constant, the capacitor voltage will have decreased to 36.8% of its initial amplitude (see Fig. 3 and Fig. 4).

If a curve of some unknown function of time is known to be exponential, a graphical technique may be employed to determine the time constant, thus enabling one to write the mathematical description of the curve. For the discharging case, consider the curve shown in Fig.3. A tangent line is drawn at any arbitrary point $y(t_o)$ on the curve. The intercept of the tangent line with the time axis yields a second point, t_b. If the time axis units are known, the time constant can be determined from $\tau = t_b - t_a$.

For a charging, or increasing exponential curve, the tangent line intercepts the asymptote to the curve. Projecting this intercept to the time axis establishes the time interval $(t_a - t_b)$ and hence τ.

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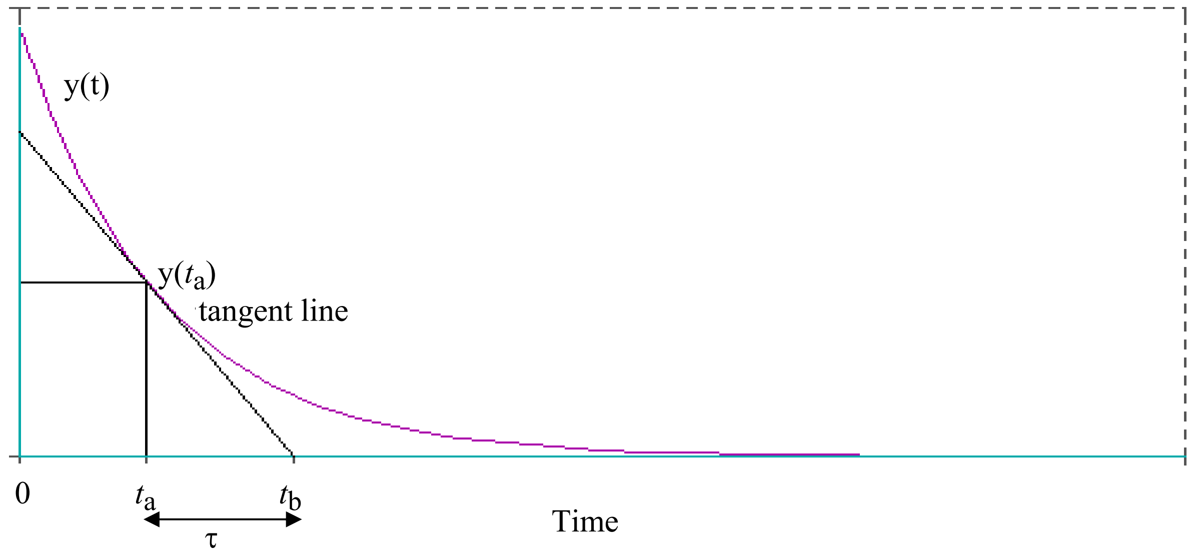

Figure 3 – Graphical determination of the time constant τ.

Case 2: The Capacitor is Discharging

Now let's consider the discharging case. Refer again to Fig. 1. Suppose that the capacitor has charged to a value V_{in} and at $t = 0$, switch S1 opens and switch S2 closes. Since the DC voltage source has been excluded, we must derive a new circuit equation. Using KCL at the same node as before

$$C\frac{dv_o(t)}{dt} + \frac{v_o}{R} = 0$$
$$Initial\ condition\colon\ v_o(0) = V_{in}$$

The solution of this linear, homogenous, constant-coefficient differential equation is

$$v_o(t) = V_{in}e^{-\frac{t}{RC}} \qquad for\ t > 0$$

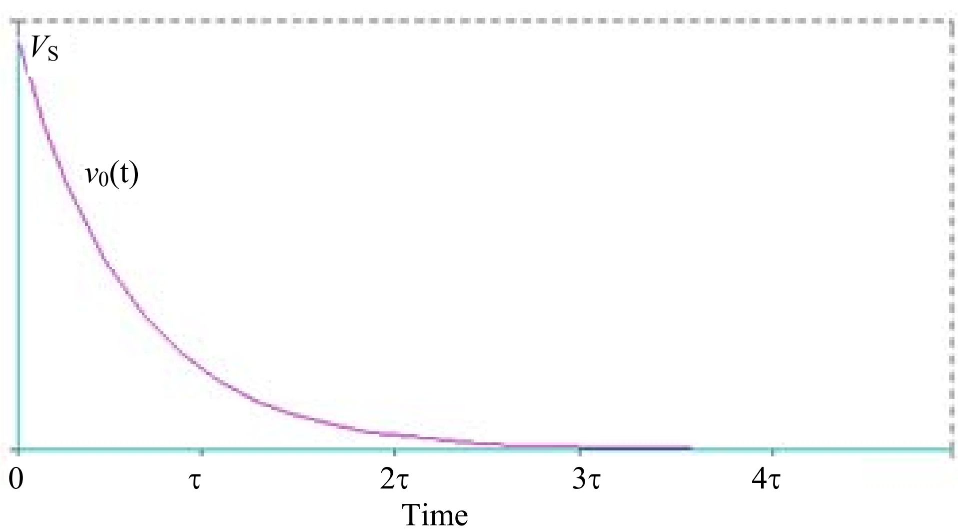

Figure 5 – Capacitor output voltage for the discharging case.

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RC Circuit Response to a Periodic Step-Voltage Excitation

With its inertia-less electron beam, the oscilloscope is particularly adapted for the display of voltage waveforms that are repetitive. The oscilloscope can continuously display some portion of a periodic input waveform. A transient waveform, however, occurs only once, and is therefore not repetitive. It can be displayed conveniently only on an oscilloscope with memory. For the oscilloscopes without memory, it is necessary to apply a repetitive "step" voltage to the input of the RC circuit to display the transient response of the circuit. A good approximation of the transient response may be obtained using a square-wave excitation since it is periodic and may be regarded as a series of positive and negative step voltages.

For a periodic square wave with a reasonably long half-period ($T/2 > 5\tau$), the exponential growth and decay during a single half-period of the square wave will be practically complete. Thus, the oscilloscope dispV_S of a periodic step voltage will appear very similar to that of a single step input to the RC circuit, as is shown in Fig. 5.

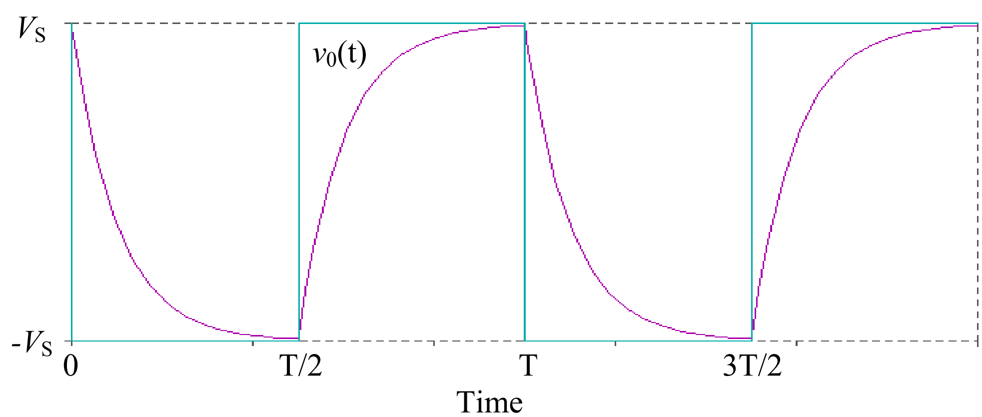

Figure 5 – Series *RC* circuit response to a "zero-centered" periodic step voltage input.

Integrating Circuit

Consider the circuit in Fig. 6. The input voltage is a pulse waveform, seen in blue, and the output voltage is in purple. Using KCL, the circuit equation can be written as

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - v_{in}(t)}{R} = 0$$

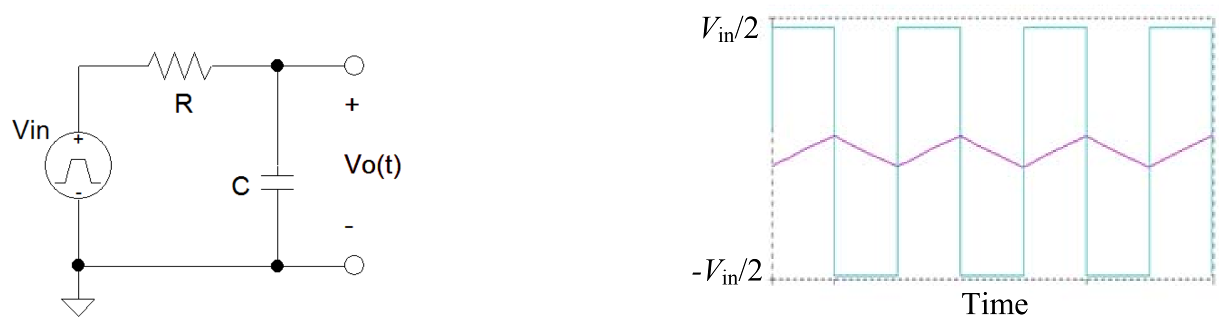

Figure 6 – Integrating circuit and a plot of input versus output voltages.

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If the time constant is very large relative to the half-period ($T/2$) of the input pulse, the circuit does not even come close to charging before the pulse falls again. In Fig. 4, the output voltage has a very low amplitude because it barely has time to charge. This allows us to make the mathematical simplification that $v_o(t) = 0$.

$$C\frac{dv_o(t)}{dt} + \frac{0 - v_{in}(t)}{R} = 0$$

$$\frac{dv_o(t)}{dt} = \frac{v_{in}(t)}{RC}$$

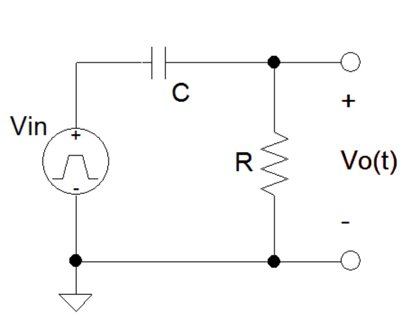
$$dv_o(t) = \frac{1}{RC} v_{in}(t)\, dt$$

$$When\ 5\tau \gg \frac{T}{2}, \qquad v_o(t) \approx \frac{1}{RC}\int v_{in}(t)dt$$

Therefore, when the condition is satisfied, the output voltage is approximately proportional to the input voltage. In layman's terms, the condition states that for the circuit in Fig. 4 to function as an integrating circuit, the charge time for the capacitor must be much greater than the duration of a single pulse.

<u>Differentiating Circuit</u>

Now consider the circuit in Fig. 7. In this circuit we have the same pulse input, but now take the output as the voltage across the resistor.



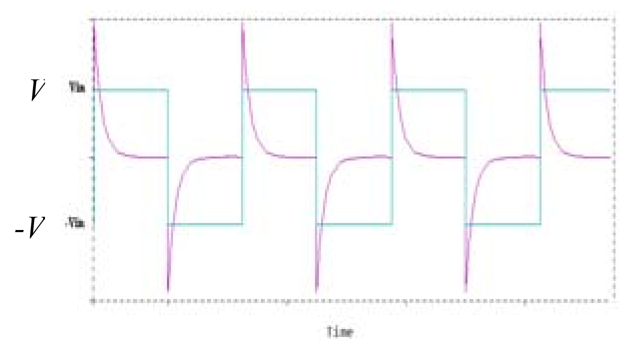


Figure 7 – Differentiating circuit and a plot of input versus output voltages.

On the rising or falling edge of the pulse, the voltage across the capacitor changes drastically. Then according to the current/voltage equation for a capacitor, the current through the capacitor will be exceedingly large. A large voltage derivative means that a large current passes through the capacitor, and so the voltage across the resistor will be high at the pulse edges. The voltage then falls off; decaying quickly if the time constant is small. It happens that when the time constant of the circuit is very small relative to the pulse half-period, the output is proportional to the derivative of the input.

$$When\ 5\tau \ll \frac{T}{2}, \qquad v_o(t) \approx RC\frac{dv_{in}(t)}{dt}$$

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In Fig. 7, we see that the output voltage can be used to approximate the derivative of the input voltage. When the input pulse is constant, the derivative is zero, the derivative is a high positive value for rising edges, and the derivative is a large negative value for falling edges.

PRE-LAB AND QUESTIONS

1. A 100 μF capacitor is connected in series with a 10 kΩ resistor and a 10 V DC source for 2 seconds. Then it is disconnected quickly and connected to a single resistor of 1 kΩ/
   a. Write the capacitor voltage, $v_c(t)$, for $0 \leq t \leq 2$ s.
   b. Write the capacitor voltage, $v_c(t')$, for $t' \geq 0$, where $t' = t - 2$.
   c. How much voltage is on the capacitor at $t = 15$ s ?

2. Show that for the discharging case, the time constant of the RC circuit is *RC*.

3. Construct the circuit of Fig. 6 in Multisim. The voltage source is the *PULSE_VOLTAGE* in the *SIGNAL_VOLTAGE_SOURCES* group.
   a. Calculate the time constant.
   b. Calculate the pulse width such that the capacitor just finishes charging by the end of the pulse. Now set the *Pulse Width* and *Period* parameters of the source (the period is twice the pulse width).
   c. Run a transient analysis for a duration equal to one period of the source. Plot the output and input voltages.
   d. Select *Cursor* > *Show Cursors*, and the two cursors will appear at the far left of your plot. Use the cursors to determine the time constant. Show your work and take a screen shot of the plot with the cursors in place.

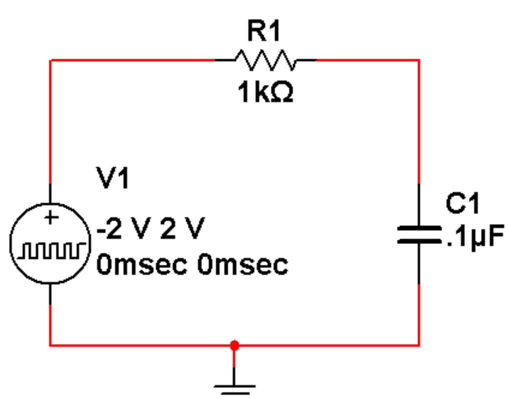

Figure 6 – Circuit for pre-lab question 2. The *PULSE_VOLTAGE* produces a periodic signal comprised of a negative pulse and a positive pulse (-2 and 2 V in this case). The pulse width parameter is the duration of the positive pulse. The period is the sum of the positive pulse and the negative pulse durations.

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LAB PROCEDURE AND QUESTIONS

1. Build the circuit of Fig. 6. The function generator (*FUNC_OUT*) will supply the pulse input. Set the frequency using the period you found in the pre-lab. Measure the output across the capacitor **and** the input with the two channels of the oscilloscope. Sketch two periods of the signals. Label the time axis.

2. Recall the condition for an integrating circuit. Choose a pulse frequency, such that the circuit of Fig. 6 satisfies this condition. Show your work. (Hint: If x >> y, then in **THIS** case, **but not in general**, x ≈ 2y will suffice.)

3. Set the function generator to this frequency, and sketch the oscilloscope output for two periods, again labeling the time axis.

4. Return the function generator to the frequency of part 1. Now choose a time constant such that the circuit satisfies the integrating circuit criterion. Show your work.

5. Select a resistance and capacitance that achieve this time constant, and replace the resistor and capacitor of Fig. 6 with these values. (Choose a capacitor from your kit. The resistor is more flexible since we have the decade resistance box.)

6. Sketch the oscilloscope output exactly as in parts 1 and 3.

7. Build now the differentiating circuit of Fig. 5. We will use the function generator, but the waveform will now be a sine wave with frequency 1 kHz. Use a capacitance of 0.01 μF and a resistance of 1 kΩ.

8. Using the cursors of the oscilloscope show that the output is approximately proportional to the derivative of the input. A well known trig identity states sin(x) = cos(x-90°), therefore since dsin(x)/dx=cos(x) the output should lag behind the input by 90°. We can convert from time delay to phase angle delay, using the formula from lab 4,

$$\Phi = \Delta T * frequency * 360°$$

   Sketch the input and output together, with the time axis labeled.