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LECTURE #7: MSOP, MPOS, K-Maps

EEL 3701: Digital Logic and Computer Systems

Based on lecture notes by Dr. Eric M. Schwartz

Definitions:

Minterm (m_i): A conjunctive/product/AND term containing one instance of every variable contained in a Boolean function. It corresponds to True (1) in only one row on an exhaustive truth table.

Maxterm (M_i): A disjunctive/sum term/OR containing one instance of every variable contained in a Boolean function. It corresponds to False (0) in only one row of an exhaustive truth table.

Sum of Products: (SOP): A disjunction of conjunctive terms (such as minterms).

Product of Sums (POS): A conjunction of disjunctive terms (such as maxterms).

Note: Any function can be written as a *sum of products* or a *product of sums*.

$$f = \sum_i \text{Minterm}_i \qquad f = \prod_i \text{Maxterm}_i$$

Example: Given a truth table, synthesize the function (F) as both SOP and POS.

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

$$F_{SOP} = \overline{A}\,\overline{B} + A\overline{B} \qquad F_{POS} = (A + \overline{B})(\overline{A} + \overline{B})$$

Does $F_{SOP} = F_{POS}$? Yes, always!

$$F_{SOP} = \overline{A}\,\overline{B} + A\overline{B} = \overline{B}$$

$$F_{POS} = (A + \overline{B})(\overline{A} + \overline{B}) = 0 + A\overline{B} + \overline{A}\,\overline{B} + \overline{B} = \overline{B}$$

Example: Given a truth table, synthesize the function (f) as both SOP and POS.

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Minterms: $f_{SOP} = \overline{a}\,\overline{b}\,\overline{c} + \overline{a}b\overline{c} + \overline{a}bc + a\overline{b}c + ab\overline{c}$

Maxterms: $f_{POS} = (\overline{a} + \overline{b} + \overline{c})(\overline{a} + b + c)(a + b + \overline{c})$

Simplification:

-Often, SOP (w/ minterms) and POS (w/ maxterms) equations can be reduced.

-The irreducible equations are called:

-*minimum sum of products* (MSOP) or

-*minimum product of sums* (MPOS)

Example: From above,

$f_{POS} = (\bar{a} + \bar{b} + \bar{c})(\bar{a} + b + c)(a + b + \bar{c})$ is the MPOS,

$f_{SOP} = \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c + ab\bar{c}$ requires some reduction.

$f_{SOP} = \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}b\bar{c} + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c + ab\bar{c}$

$f_{SOP} = \bar{a}b + \bar{a}\bar{c} + a\bar{b}c + b\bar{c}$ is the MSOP.

=> Which takes less gates? Try them both and find out.

Boolean Algebra Theorems and Duality:

-See "Laws and Theorems of Boolean Algebra" on website

Useful Theorems for Simplification:

$XY + X\bar{Y} = X \quad \Leftrightarrow \quad (X+Y)(X+\bar{Y}) = X$

$X + XY = X \quad \Leftrightarrow \quad X(X+Y) = X$

$(X+\bar{Y})Y = XY \quad \Leftrightarrow \quad X\bar{Y} + Y = X + Y$

=> These are all special cases of the distributive law:

$X(Y+Z) = XY + XZ \quad \Leftrightarrow \quad X + YZ = (X+Y)(X+Z)$

Consensus Theorem: $XY + YZ + \bar{X}Z = XY + \bar{X}Z$

Proof: $XY + YZ + \bar{X}Z = XY + (X + \bar{X})YZ + \bar{X}Z$

$= XY + XYZ + \bar{X}YZ + \bar{X}Z$

$= XY + XYZ + \bar{X}Z + \bar{X}YZ$

$= XY(1+Z) + \bar{X}Z(1+Y)$

$= XY + \bar{X}Z$ Q.E.D.

Duality is helpful:

Example: $(A+B+C)(A+B+\bar{C}) = ???$

Dual: $ABC + AB\bar{C} = AB$

Therefore: $(A+B+C)(A+B+\bar{C}) = A+B$

Karnaugh Maps (K-Maps):

-Two terms which differ in only one literal can be reduced by that literal

=> $XY + X\bar{Y} = X \quad \Leftrightarrow \quad (X+Y)(X+\bar{Y}) = X$

-K-maps are a method to aid humans in the generating MSOP or MPOS.

=> K-maps graphically enable a human to perform the above equations.

3 Variable Truth Table

a	b	c	f(a,b,c)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

=> $f_{SOP} = \overline{a}\,\overline{b}\,\overline{c} + \overline{a}b\overline{c} + \overline{a}bc + a\overline{b}c + ab\overline{c} = \overline{a}b + \overline{a}\,\overline{c} + a\overline{b}c + b\overline{c}$

3 Variable K-Map

ab \c	0	1
00	1	0
01	1	1
11	1	0
10	0	1

=> Notice that only 1 literal changes from square to square.
=> By circling adjacent implicants, we can directly obtain:

$$f_{SOP} = \overline{a}b + \overline{a}\,\overline{c} + a\overline{b}c + b\overline{c}$$

Definitions:

Implicants: Terms in a K-map that are True (1) iff the function is True (1).
Prime Implicants: Implicants that are as general as possible (cannot be reduced).
Essential Prime Implicant: A prime implicant that covers a 1 that no other prime implicant covers.

Solving MSOP Using K-Maps:

Process: 1) Circle the prime implicants.
2) Select a subset that covers the function.

Note: Implicants are formed by clusters of 2^n (1, 2, 4, 8, 16…).

Example: Simplify $g = \overline{a}\,\overline{b}\,\overline{c} + \overline{a}\,\overline{b}c + \overline{a}b\overline{c} + ab\overline{c} + a\overline{b}c$ to MSOP form.

K-Map

ab \c	0	1
00	1	1
01	1	0
11	1	0
10	0	1

$$f_{MSOP} = \overline{a}\,\overline{b} + \overline{b}c + b\overline{c}$$

OR $f_{MSOP} = \overline{a}\,\overline{c} + \overline{b}c + b\overline{c}$

Solving MPOS Using K-Maps:

-We can also solve for POS by following the same process with 0's.

-Reminder: POS is solved to make the equation true everywhere BUT on the 0.

This means that you must "flip" the complement such that:

1's => Complemented variables

0's => Uncomplemented variables

Example: Simplify $g = \overline{a}\overline{b}\overline{c} + \overline{a}\overline{b}c + \overline{a}b\overline{c} + ab\overline{c} + a\overline{b}c$ to MPOS form.

K-Map

ab \c	0	1
00	1	1
01	1	0
11	1	0
10	0	1

$$f_{MPOS} = (\overline{a} + b + c)(\overline{b} + \overline{c})$$

Which is better $f_{MSOP} = \overline{a}\overline{b} + \overline{b}c + b\overline{c}$ OR $f_{MPOS} = (\overline{a} + b + c)(\overline{b} + \overline{c})$?

-MSOP => 4 gates, 9 inputs -MPOS => 3 gates, 7 inputs

-Depends on your design criteria.

-Heuristic (rule of thumb): In general, use whatever has the least, 0's or 1's.

-Must do both and compare to know for sure.

Observation: K-maps give us 2-level logic such that the maximum propagation delay is 2 gates. Sometimes, it is possible to utilize a more logic levels to save gates, but the delay will be longer. (Note that 2-level logic is the fastest logic possible.)

Examples with 4 Inputs:

Example #1: Determine the MSOP for the following K-map.

ab\cd	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	1	0	0	0
10	1	1	0	1

$$f_{MSOP} = a\overline{b}\overline{c} + a\overline{b}\overline{d} + a\overline{c}\overline{d} + \overline{b}\overline{c}\overline{d}$$

Example #2: Determine the MSOP for the following K-map.

ab\cd	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

$$f_{MSOP} = \overline{a}bc + \overline{a}\overline{c}d + ab\overline{c} + acd$$

Note: Term bd is redundant, so it was left out.

Example #3: Determine the MSOP for the following K-map.

ab\cd	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	1	1	0
10	0	0	0	0

$$f_{MSOP} = \overline{a}b + bd$$

Example #4: Determine the MSOP and MPOS for the following K-map.

ab\cd	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

$$f_{MSOP} = \overline{b}d + b\overline{d} = b \oplus d$$

$$f_{MPOS} = (\overline{b} + \overline{d})(b + d) = b \oplus d$$

"Don't Care" Values:

-Some systems do not use every possible combination of every input.

-We handle these inputs using "don't care" values of X.

-X can equal 0 or 1, whatever is most convenient for reducing equations.

-X cannot equal both 0 and 1 in the same design.

Example: A 4-input system where combos 0101, 1101, and 1011 are not used.

ab\cd	00	01	11	10
00	0	0	1	1
01	1	X	0	1
11	1	X	0	0
10	0	0	X	0

$$f_{MSOP} = \overline{a}\overline{b}c + \overline{a}c\overline{d} + b\overline{c}$$

$$f_{MPOS} = (\overline{a} + \overline{c})(\overline{b} + \overline{d})(b + c)$$

Note: When using "don't cares," f_{MSOP} does not necessarily equal f_{MPOS}.

K-Maps with 5 or 6 Variables:

=> Create multiple 4-variable K-maps with static conditions for the 5^{th} (and 6^{th}) variable.

Example: A 5 variable K-map with inputs a, b, c, d, and e.

e = 0

ab\cd	00	01	11	10
00	0	1	1	0
01	0	0	0	1
11	0	0	0	1
10	0	1	1	0

e = 1

ab\cd	00	01	11	10
00	0	0	0	0
01	1	1	0	1
11	0	0	0	1
10	0	1	0	0

$$f_{MSOP} = \overline{a}b\overline{c}e + a\overline{b}\overline{c}d + \overline{b}d\overline{e} + bc\overline{d}$$