## Analysis of Cross-View IR Sensors

## Introduction

Two IR emitter-detector proximity sensors are mounted on a mobile robot platform. The left sensor looks to the right and the right sensor looks to the left, hence, the two IR sensors perceive cross fields of view. The following analysis computes the differential change in the two IR ranges as the robot approaches a boundary of an object.

## Geometric Analysis of Cross-View Sensors

The mobile robot approaches a wall at angle $\psi$ (refer to Fig. 1), moving $v d t$ towards the wall at that heading. The left sensor direction forms an angle $\theta_{L}$ to the midline of the robot, along the axis of the two center drive axis. The right sensor direction forms and angle $\theta_{R}$ with respect to the same axis. The beams cross a few centimeters in front of the robot. The problem is to compute $d_{R}$ and $d_{L}$ in terms of $v d t$ and the angles shown.

Since corresponding angles of paralle lines are equal, the angle between $d_{L}$ and the displacement $v d t$ equals $90^{\circ}-\theta_{L}$. The angle opposite the side of length $v d t$ equals $\psi$.


Figure 1. A robot approaching a wall.
By the law of sines, obtain an expression for the change in the left sensor range $d_{L}$,

$$
\frac{d_{L}}{\sin (\psi)}=\frac{v \cdot d t}{\sin \left(90^{\circ}+\theta_{\mathrm{L}}-\psi\right)}
$$

In the triangle containing $d_{R}$ the law of sines yields

$$
\frac{d_{R}}{\sin \left(180^{\circ}-\psi\right)}=\frac{v \cdot d t}{\sin \left(-90^{\circ}+\theta_{\mathrm{R}}+\psi\right)} .
$$

The two previous expressions lead to

$$
d_{L}=\frac{v \cdot d t \cdot \sin (\psi)}{\cos \left(\theta_{\mathrm{L}}-\psi\right)}
$$

and

$$
d_{R}=\frac{-v \cdot d t \cdot \sin (\psi)}{\cos \left(\theta_{\mathrm{R}}+\psi\right)}
$$

The angle definitions in the figure do not share a common reference. To effectively demonstrate the relationship between the two expressions, reference the angles to an $x$-axis defined as horizontal and pointing to the right of the page. The mapping of the angles to this reference is

$$
\theta_{L} \rightarrow \theta_{L} ; \quad \theta_{R} \rightarrow 180^{\circ}-\theta_{R} ; \quad \psi \rightarrow 270^{\circ}+\psi
$$

With these definitions of the angles, the solutions to the changes in the left and right IR sensor ranging becomes,

$$
d_{L}=\frac{v \cdot d t \cdot \cos (\psi)}{\sin \left(\theta_{\mathrm{L}}-\psi\right)} \quad(\text { Angles defined wrt } x \text {-axis) }
$$

and

$$
d_{R}=\frac{v \cdot d t \cdot \cos (\psi)}{\sin \left(\theta_{\mathrm{R}}-\psi\right)} \quad(\text { Angles defined wrt } x \text {-axis })
$$

Of course, the principal value of the two different angle definitions are the same, so the expressions only look different. The new expression enables us to recognize both the inherent symmetry of the equations and to discuss the relations for all possible arguments with, perhaps, added clarity. In particular, observe that the differential changes are independent of $\psi$ when $\theta_{L}=180^{\circ}-\theta_{R}=90^{\circ}$. An issue is whether crossing the eyes ( $\theta_{L}=180^{\circ}-\theta_{R} \neq 90^{\circ}$ ), which makes the sensor readings sensitive to the angle of approach $\psi$, adds any collision avoidance features.

## Advantage of Crossed-View Sensors

If the sensors point straight ahead $\left(\theta_{L}=180^{\circ}-\theta_{R}=90^{\circ}\right.$ ), then the differential change is always the same and small objects between the sensors cannot be detected. In contrast, cross-view sensors can detect small objects as they approach the center of the robot. The cross-view robot is least sensitive as it approaches an object at $\psi=O^{\circ}$. As the approach angle $\psi$ increases

When $\sin \left(\theta_{\mathrm{R}}-\psi\right)=0, \psi=\theta_{R}+k \pi$, the expression for $d_{R}$ becomes singular, indicating that the wall is parallel to the line of sight of the right sensor. A similar observation applies to the left sensor. Neither sensor can view the wall if the robot heading $\psi$ is outside of the range $-180^{\circ}+\theta_{L}<\psi<\theta_{R}$. For $\psi$ outside of this range the wall is is headed away from the wall and neither sensor experiences a change in reading due to that particular wall.

In the last two equations, the newly defined $\psi$ equals zero when the wall is horizontal. The displacement $d_{L}$ and $d_{R}$ become equal if the sensors are designed so that $\theta_{R}$ $=180^{\circ}-\theta_{L}$. Otherwise, one sensor will yield a larger reading than the other when the robot motion heads perpendicular to the wall $(\psi=0)$, a situation where the two sensors should read the same. Observe that the displacements do not increase as the robot approaches the wall, regardless of the approach angle $\psi$. Since it may be desirable to increase the robots reaction the closer it approaches the wall, dividing the
change by the range to the wall will enhance its reactivity as it approaches the wall. This will be discussed later.

At $\psi=0$ the sensor response will be larger for small $\theta_{R}$ and $\theta_{L}$, but less sensitive as $\psi$ increases or decreases. But, from the range of the equation, the base line $b$ of the robot, the length of the wall must be large in order for the robot to see it. Further, the distance to the wall on a direct heading will be extremely short once it is detected. Neither situatation is desirable. Suppose we specify that the robot should detect an object of "zero" width at $d_{c}$. Assuming the sensor angles have the same principal value, the line of sight of the two sensor will intersect at $d_{c}$ from their base lines, hence,

$$
\theta_{L}=\tan ^{-1}\left(2 d_{c} / b\right)=180^{\circ}-\theta_{R}
$$

When $\psi=90^{\circ}$ the robot runs parallel to the wall and neither sensor register's a change. If the robot should deviate towards the wall

