EEL3135: Homework #1

(10 problems, distributed 1/30/2002, due 2/11/2002)

Instructions:

Show/explain all work to get full credit.

Problem 1:

(a) Explain the difference between a continuous-time and discrete-time signal.

(b) Give at least one reason why the study of each type of signal (continuous-time and discrete-time) is important.

Problem 2:

(a) Sketch the magnitude spectrum (frequency-domain representation) for the following continuous-time signal:

$$x(t) = 1 + 4\cos(30t) + 3\cos(60\pi t) \quad (1)$$

Be sure to label your plot. [Note: It is not important that you indicate absolute magnitude on your plot, only the relative magnitude of different frequency components.]

(b) Sketch the magnitude spectrum for the following continuous-time signal:

$$x(t) = \cos(4\pi t)\cos(20\pi t) \quad (2)$$

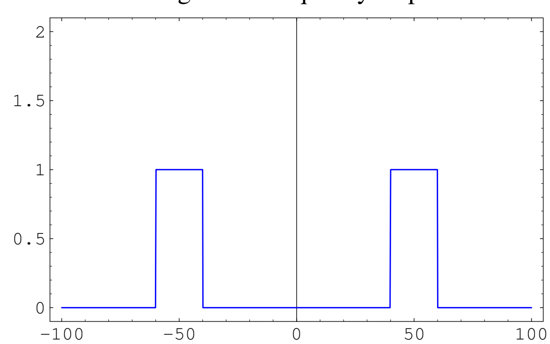
Hint: The following trigonometric identity may be useful:

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha + \beta) + \frac{1}{2}\cos(\alpha - \beta) \quad (3)$$

Problem 3:

For the continuous-time signal $x(t) = 1 + \cos(2\pi \cdot 50t) + \cos(2\pi \cdot 80t)$, explain how the signal would be changed by an ideal filter, whose frequency response is plotted below.

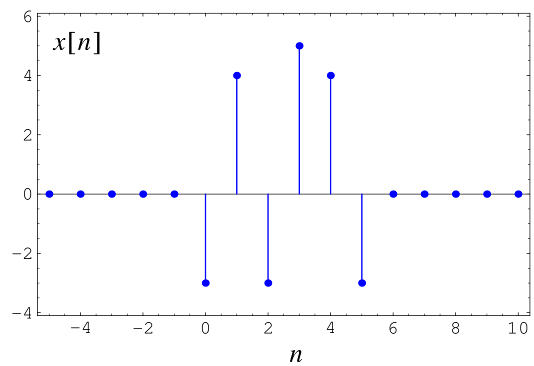
filter magnitude frequency response



f (frequency)

Problem 4:

(a) Compute the output sequence $y[n]$, $n \geq 0$, for the difference equation $y[n] = x[n] - 3x[n-1]$ and the input sequence $x[n]$ plotted below.



(b) Repeat part (a) for the following difference equation:

$$y[n] = -y[n-1] + 2x[n] - x[n-1] \tag{4}$$

and $0 \leq n \leq 10$. Assume $y[-1] = 0$.

Problem 5:

Assume you want to sample and filter the continuous-time signal $x(t) = \sin(2\pi t)$; further assume that the discrete-time filter you want to apply to the sampled signal $x[n]$ is given by the difference equation below:

$$y[n] = \frac{1}{5}x[n] + \frac{1}{5}x[n-1] + \frac{1}{5}x[n-2] + \frac{1}{5}x[n-3] + \frac{1}{5}x[n-4] \tag{5}$$

For which of the following sampling frequencies (samples/second) —1Hz, 3Hz, 5Hz and 7Hz —will the output $y[n]$ be zero for all n?

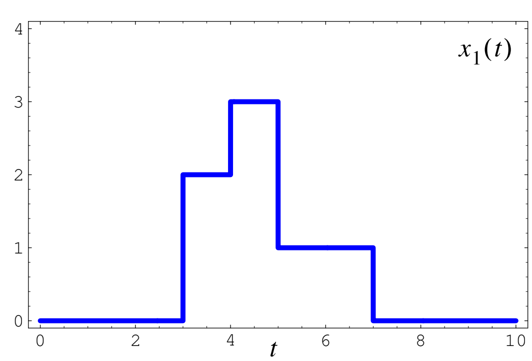
Problem 6:

Consider the continuous-time function $x_1(t)$ below, which is nonzero in the interval $t \in [3, 7)$.

(a) Sketch the following related functions: $x_1(-t)$, $x_1(t-2)$, $x_1(t+4)$, $2x_1(t) + 1$, $x_1(2t-4)$, $x_1(-t+2)$. Be sure to label all critical points on your plots.

(b) Write an analytic expression for $x_1(t)$.

(c) Write analytic expressions for each of the functions in part (a).



Problem 7:

Sketch the following functions. Be sure to label all critical points on your plot.

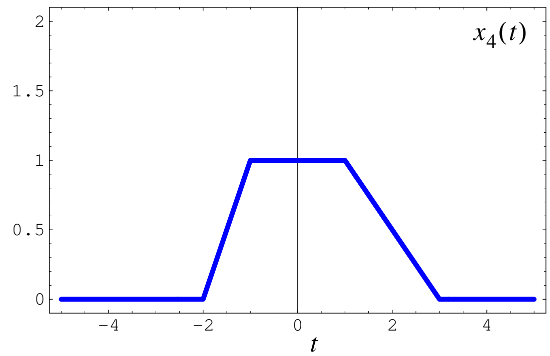
(a) $x_2(t) = 3\cos(4\pi t)[u(t) - u(t-1)] + 1$

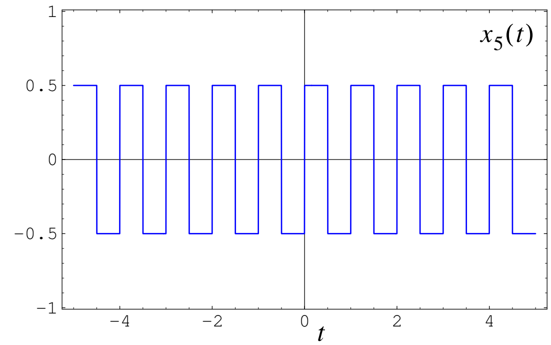
(b) $x_3(t) = -(t+3)u(t+3) + tu(t) + (t-1)u(t-1) - (t-5)u(t-5)$

Problem 8:

(a) Give an analytic expression for the function $x_4(t)$, which is nonzero in the interval $t \in (-2, 3)$.

(b) Give an analytic expression for the periodic square wave $x_5(t)$.





Problem 9:

For this problem, assume the sampling formula,

$$x[n] = x_c(n/f_s) \tag{6}$$

where f_s denotes the sampling frequency in Hz (samples/second), $x[n]$ is a discrete-time function, and $x_c(t)$ represents some continuous-time function.

(a) Let $f_s = 3$ Hz. Plot $x_1[n]$ corresponding to the continuous-time signal $x_1(t)$ (Problem 6). In other words,

$$x_1[n] = x_1(n/f_s). \tag{7}$$

(b) Repeat part (a) for $f_s = 5$ Hz.

Problem 10:

Evaluate the following integrals for the continuous-time signals previously defined:

$$\int_{-\infty}^{\infty} \delta(t-2)x_1(t+4)dt \tag{8}$$

$$\int_{-\infty}^{\infty} \delta(t)x_4(t-2)dt \tag{9}$$