EEL3135: Homework #1 Solutions

Problem 1:

(a) Explain the difference between a continuous-time and discrete-time signal.

A continuous-time signal $x(t)$ is a continuous functions of time t, a real-valued variable, while a discrete-time signal $x[n]$ is a function of time index n, an integer-valued variable; that is, a discrete-time signal is only defined for a discrete number of points. Oftentimes, discrete-time signals are sampled versions of continuous-time signals.

(b) Give at least one reason why the study of each type of signal (continuous-time and discrete-time) is important.

Continuous-time signals exist in the real-world, and any discrete-time processing system (e.g. computer) must interface with continuous-time signals both at the input and the output. Discrete-time signals have grown in importance with the increased use of computers as digital signal processing (DSP) systems. As such, both types of signals are important in understanding a complete signal processing system.

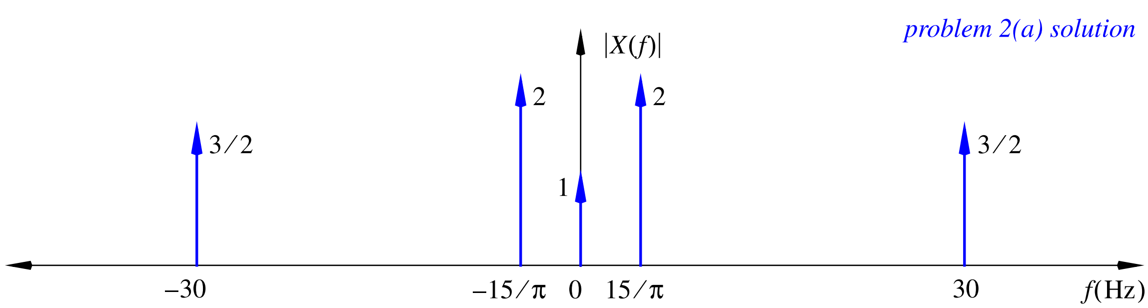
Problem 2:

(a) Sketch the magnitude spectrum (frequency-domain representation) for the following continuous-time signal:

$$x(t) = 1 + 4\cos(30t) + 3\cos(60\pi t) \quad (1)$$

Be sure to label your plot. [Note: It is not important that you indicate absolute magnitude on your plot, only the relative magnitude of different frequency components.]

The frequencies in $x(t)$ occur at $\pm 30/2\pi = \pm 15/\pi$ Hz and $\pm 60\pi/2\pi = \pm 30$ Hz (cycles/second).



(b) Sketch the magnitude spectrum for the following continuous-time signal:

$$x(t) = \cos(4\pi t)\cos(20\pi t) \quad (2)$$

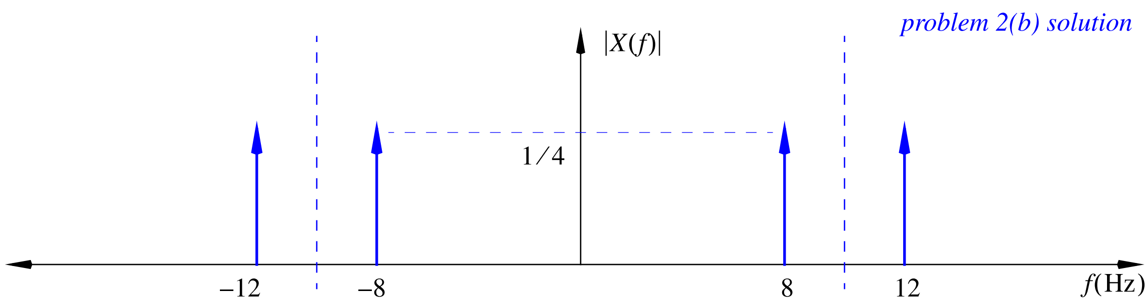
Hint: The following trigonometric identity may be useful:

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta) \quad (3)$$

Using identity (3), we can rewrite equation (2) as:

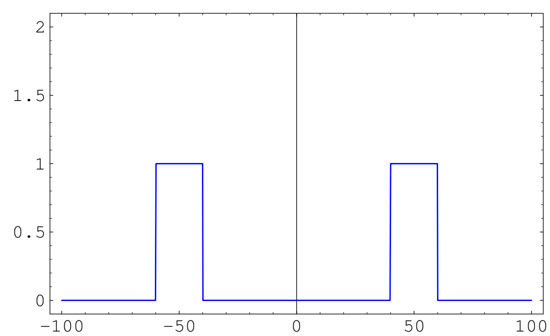
$$\begin{aligned} x(t) &= \frac{1}{2}\cos(20\pi t + 4\pi t) + \frac{1}{2}\cos(20\pi t - 4\pi t) \\ &= \frac{1}{2}\cos(24\pi t) + \frac{1}{2}\cos(16\pi t) \end{aligned} \quad \text{(S-1)}$$

The frequencies in $x(t)$ occur at $\pm 24\pi/2\pi = \pm 12$ Hz and $\pm 16\pi/2\pi = \pm 8$ Hz (cycles/second). The magnitude spectrum for equation (S-1) is plotted on top of page 2.



Problem 3:

For the continuous-time signal $x(t) = 1 + \cos(2\pi \cdot 50t) + \cos(2\pi \cdot 80t)$, explain how the signal would be changed by an ideal filter, whose frequency response is plotted below.



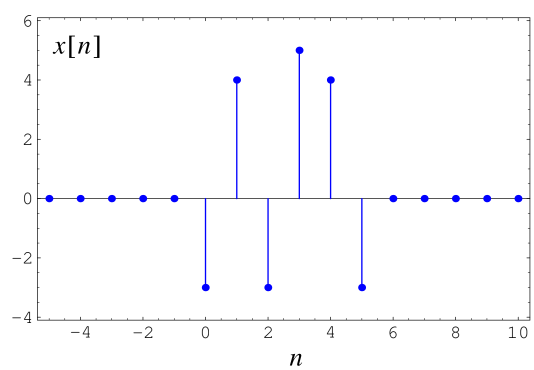
After the signal $x(t)$ is filtered by the above ideal filter, the output $y(t)$ will be given by,

$$y(t) = \cos(2\pi \cdot 50t + \alpha) \quad \text{(S-2)}$$

(Not enough information is given to determine α.) That is the DC component and 80Hz component will be completely eliminated.

Problem 4:

(a) Compute the output sequence $y[n]$, $n \geq 0$, for the difference equation $y[n] = x[n] - 3x[n-1]$ and the input sequence $x[n]$ plotted below.



$$y[0] = x[0] - 3x[-1] = -3 \quad \text{(S-3)}$$

$$y[1] = x[1] - 3x[0] = 13 \quad \text{(S-4)}$$

$$y[2] = x[2] - 3x[1] = -15 \quad \text{(S-5)}$$

$$y[3] = x[3] - 3x[2] = 14 \quad \text{(S-6)}$$

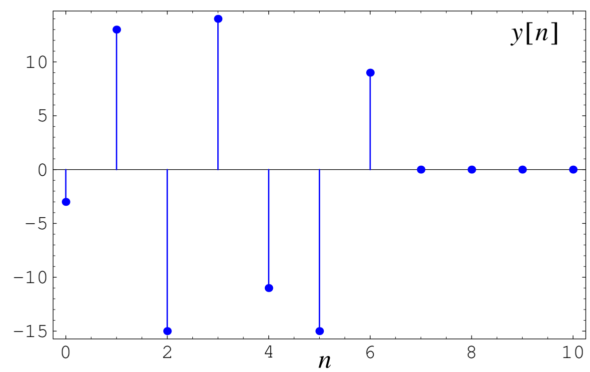
$$y[4] = x[4] - 3x[3] = -11 \quad \text{(S-7)}$$

$$y[5] = x[5] - 3x[4] = -15 \quad \text{(S-8)}$$

$$y[6] = x[6] - 3x[5] = 9 \quad \text{(S-9)}$$

$$y[n] = 0,\ n \geq 7. \quad \text{(S-10)}$$

This output sequence $y[n]$ is plotted on top of the next page.



(b) Repeat part (a) for the following difference equation:

$$y[n] = -y[n-1] + 2x[n] - x[n-1] \quad (4)$$

and $0 \leq n \leq 10$. Assume $y[-1] = 0$.

$$y[0] = -y[-1] + 2x[0] - x[-1] = -6 \quad \text{(S-11)}$$

$$y[1] = -y[0] + 2x[1] - x[0] = 17 \quad \text{(S-12)}$$

$$y[2] = -y[1] + 2x[2] - x[1] = -27 \quad \text{(S-13)}$$

$$y[3] = -y[2] + 2x[3] - x[2] = 40 \quad \text{(S-14)}$$

$$y[4] = -y[3] + 2x[4] - x[3] = -37 \quad \text{(S-15)}$$

$$y[5] = -y[4] + 2x[5] - x[4] = 27 \quad \text{(S-16)}$$

$$y[6] = -y[5] + 2x[6] - x[5] = -24 \ \text{(note } x[n] = 0,\ n > 5\text{)} \quad \text{(S-17)}$$

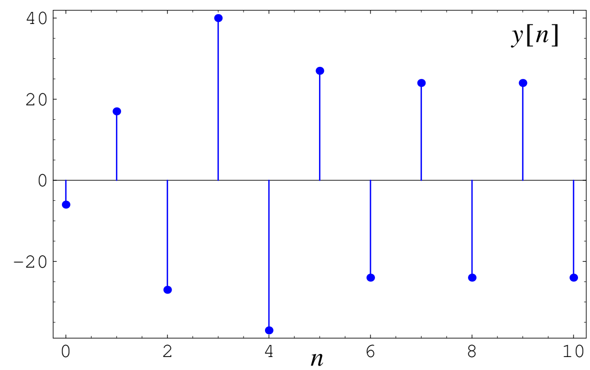
$$y[7] = -y[6] + 2x[7] - x[6] = 24 \quad \text{(S-18)}$$

$$y[8] = -y[7] + 2x[8] - x[7] = -24 \quad \text{(S-19)}$$

$$y[9] = -y[8] + x[9] - x[8] = 24 \quad \text{(S-20)}$$

$$y[10] = -y[9] + 2x[10] - x[9] = -24 \quad \text{(S-21)}$$

This output sequence $y[n]$ is plotted on the top of the next page.



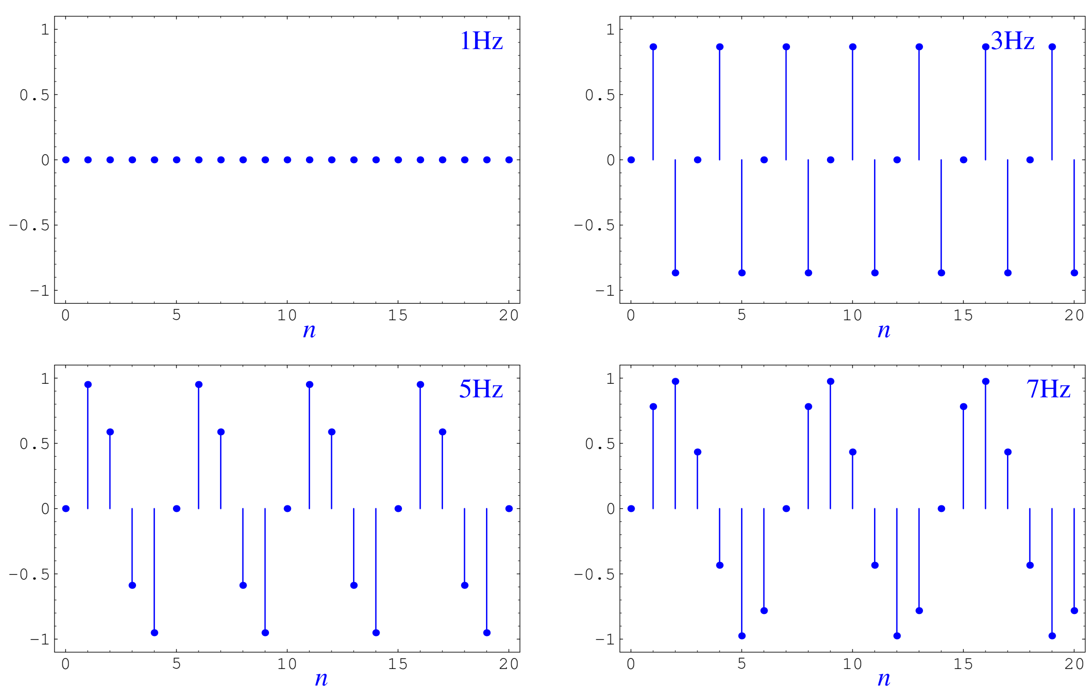
Problem 5:

Assume you want to sample and filter the continuous-time signal $x(t) = \sin(2\pi t)$; further assume that the discrete-time filter you want to apply to the sampled signal $x[n]$ is given by the difference equation below:

$$y[n] = \frac{1}{5}x[n] + \frac{1}{5}x[n-1] + \frac{1}{5}x[n-2] + \frac{1}{5}x[n-3] + \frac{1}{5}x[n-4] \qquad (5)$$

For which of the following sampling frequencies (samples/second) —1Hz, 3Hz, 5Hz and 7Hz —will the output $y[n]$ be zero for all n?

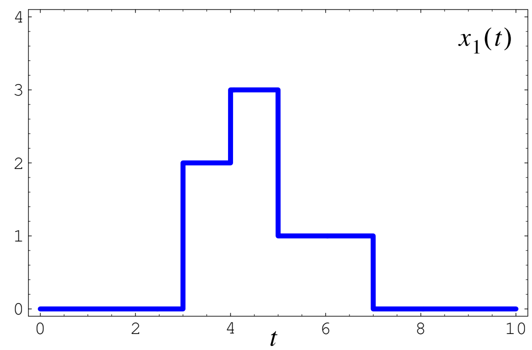
The plot below shows the sequences that would be generated at the four different sampling frequencies. Note that the filter in equation (5) will output zero for all n, if five consecutive samples of the input sequence average to zero. This is the case for the 1Hz and 5Hz sampling frequencies, and is not true for the 3Hz and 7Hz sampling frequencies. Therefore, $y[n]$ will be zero for sampling frequencies 1Hz and 5Hz.[1]



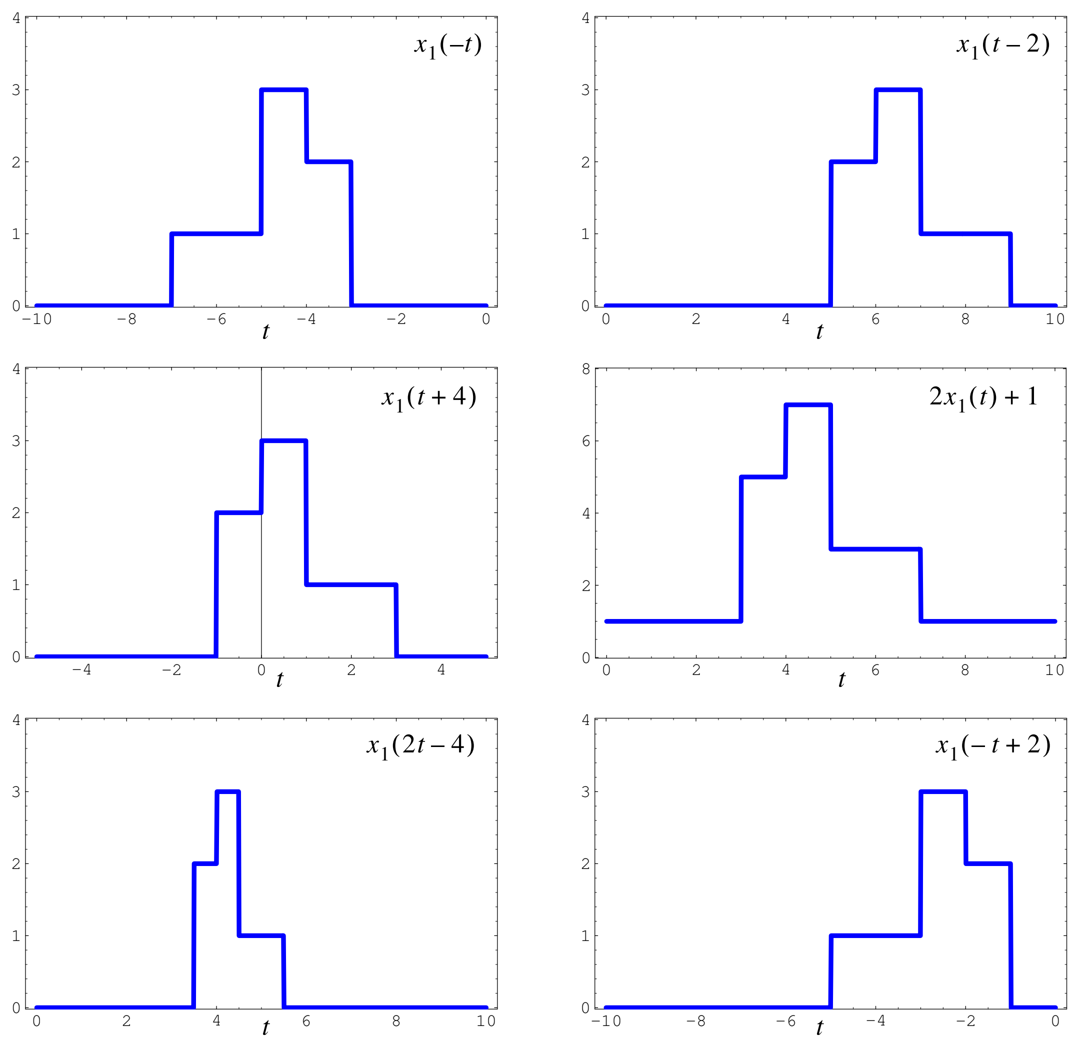
1. Note that in general, the 1Hz sampling frequency will not result in zero output, since we could easily offset the sampling so that we would get a constant sequence of samples not equal to zero.

Problem 6:

Consider the continuous-time function $x_1(t)$ below, which is nonzero in the interval $t \in [3, 7)$.



(a) Sketch the following related functions: $x_1(-t)$, $x_1(t-2)$, $x_1(t+4)$, $2x_1(t)+1$, $x_1(2t-4)$, $x_1(-t+2)$. Be sure to label all critical points on your plots.



(b) Write an analytic expression for $x_1(t)$.

$$x_1(t) = 2u(t-3) + u(t-4) - 2u(t-5) - u(t-7) \tag{S-22}$$

(c) Write analytic expressions for each of the functions in part (a).

$$x_1(-t) = 2u(-t-3) + u(-t-4) - 2u(-t-5) - u(-t-7) \quad \text{(S-23)}$$

$$x_1(t-2) = 2u(t-5) + u(t-6) - 2u(t-7) - u(t-9) \quad \text{(S-24)}$$

$$x_1(t+4) = 2u(t+1) + u(t) - 2u(t-1) - u(t-3) \quad \text{(S-25)}$$

$$2x_1(t) + 1 = 4u(t-3) + 2u(t-4) - 4u(t-5) - 2u(t-7) + 1 \quad \text{(S-26)}$$

$$x_1(2t-4) = 2u(2t-7) + u(2t-8) - 2u(2t-9) - u(2t-11) \quad \text{(S-27)}$$

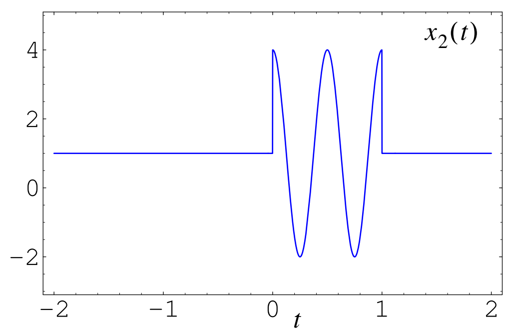
$$x_1(-t+2) = 2u(-t-1) + u(-t-2) - 2u(-t-3) - u(-t-5) \quad \text{(S-28)}$$

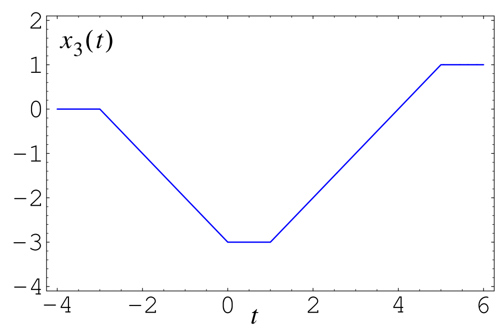
Problem 7:

Sketch the following functions. Be sure to label all critical points on your plot.

(a) $x_2(t) = 3\cos(4\pi t)[u(t) - u(t-1)] + 1$

(b) $x_3(t) = -(t+3)u(t+3) + tu(t) + (t-1)u(t-1) - (t-5)u(t-5)$;



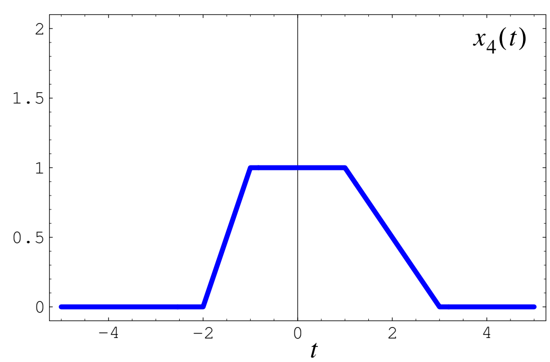


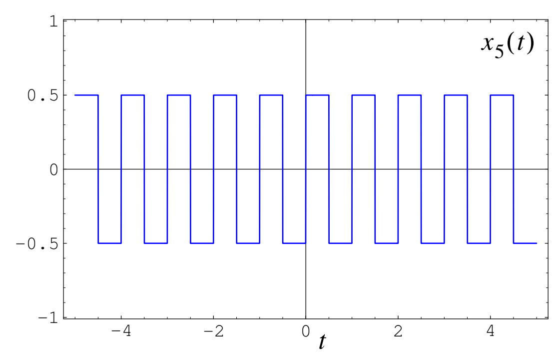
Problem 8:

(a) Give an analytic expression for the function $x_4(t)$, which is nonzero in the interval $t \in (-2, 3)$.

$$x_4(t) = (t+2)[u(t+2) - u(t+1)] + [u(t+1) - u(t-1)] + (-t/2 + 3/2)[u(t-1) - u(t-3)] \quad \text{(S-29)}$$

(b) Give an analytic expression for the periodic square wave $x_5(t)$.





Let us first define one cycle for $0 \le t < 1$:

$$c(t) = (1/2)u(t) - u(t-1/2) + (1/2)u(t-1) \quad \text{(S-30)}$$

Then $x_5(t)$ is given by,

$$x_5(t) = \sum_{k=-\infty}^{\infty} c(t-k) \quad \text{(S-31)}$$

Problem 9:

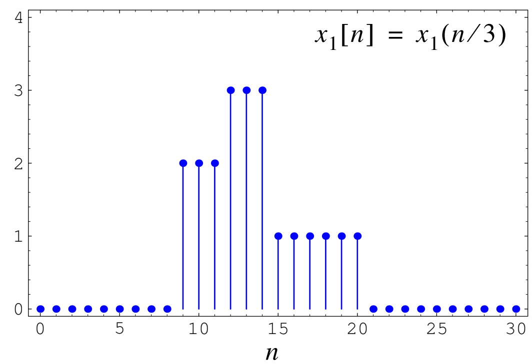
For this problem, assume the sampling formula,

$$x[n] = x_c(n/f_s) \quad (6)$$

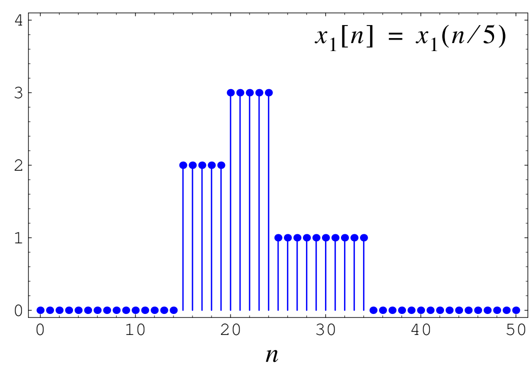
where f_s denotes the sampling frequency in Hz (samples/second), $x[n]$ is a discrete-time function, and $x_c(t)$ represents some continuous-time function.

(a) Let $f_s = 3$ Hz. Plot $x_1[n]$ corresponding to the continuous-time signal $x_1(t)$ (Problem 6). In other words,

$$x_1[n] = x_1(n/f_s). \quad (7)$$



(b) Repeat part (a) for $f_s = 5$ Hz.



Problem 10:

Evaluate the following integrals for the continuous-time signals previously defined:

$$\int_{-\infty}^{\infty} \delta(t-2)x_1(t+4)dt = x_1(2+4) = x_1(6) = 1 \quad (8)$$

$$\int_{-\infty}^{\infty} \delta(t)x_4(t-2)dt = x_4(0-2) = x_4(-2) = 0 \quad (9)$$