EEL3135: Homework #2

(9 problems, distributed 2/11/2002, due 2/21/2002)

Instructions:

Show/explain all work to get full credit. You may use mathematical software to arrive at your solutions; however, be sure that if you do, (1) you turn in and explain the code used to generate your answer, and (2) you understand how to generate the answer by hand as well, since you will not have the aid of a computer during exams.

Problem 1:

Consider the continuous-time signal:

$$x_1(t) = \cos(12\pi t) . \tag{1}$$

Assume that you sample $x_1(t)$ such that,

$$x_1[n] = x_1(n/f_s) \tag{2}$$

where $f_s = 5$ Hz, and then pass the sampled sequence $x_1[n]$ through an ideal low-pass filter with cut-off frequencies at $\pm f_s/2 = \pm 2.5$ Hz to produce the signal $x_r(t)$.

(a) Give an analytic expression for $x_r(t)$. Be sure to explain all the steps required to get your answer.

(b) Plot $x_r(n/f_s)$ and $x_1(n/f_s)$ for $n \in \{0, 1, \ldots, 9, 10\}$.

(c) Give analytic expressions for at least two aliases of $x_1(t)$ — that is, continuous-time signals not equal to $x_1(t)$ that will yield the same discrete-time sequences as $x_1[n]$ for $f_s = 5$ Hz.

(d) What is the Nyquist sampling frequency corresponding to signal $x_1(t)$?

Problem 2:

Consider the continuous-time signal:

$$x_2(t) = 1 + 2\cos(2\pi t) + 3\cos(4\pi t) . \tag{3}$$

Assume that you sample $x_2(t)$ such that,

$$x_2[n] = x_2(n/f_s) \tag{4}$$

where $f_s = 3$ Hz, and then pass the sampled sequence $x_2[n]$ through an ideal low-pass filter with cut-off frequencies at $\pm f_s/2 = \pm 1.5$ Hz to produce the signal $x_r(t)$.

(a) Give an analytic expression for $x_r(t)$. Be sure to explain all the steps required to get your answer.

(b) What is the Nyquist sampling frequency corresponding to signal $x_2(t)$?

Problem 3:

Let the frequency representation $X_3(f)$ of a continuous-time signal $x_3(t)$ be given by,

$$X_3(f) = |f|\,[u(f+f_{max}) - u(f-f_{max})] . \tag{5}$$

Assume that you sample $x_3(t)$ such that,

$$x_3[n] = x_3(n/f_s) , \tag{6}$$

and then pass the sampled sequence $x_3[n]$ through an ideal low-pass filter with cut-off frequencies at $\pm f_s/2$ to produce the signal $x_r(t)$.

(a) Plot the frequency representation $X_r(f)$ of the signal $x_r(t)$ for the sampling frequency $f_s = 3f_{max}$, and give an analytic expression for $X_r(f)$

(b) Plot the frequency representation $X_r(f)$ of the signal $x_r(t)$ for the sampling frequency $f_s = 2f_{max}/3$, and give an analytic expression for $X_r(f)$.

Problem 4:

Assume that you sampled a piece of music at sampling frequency $f_s > 2f_{max}$ but encoded an incorrect sampling frequency in the resulting digital music file (e.g. wav or mp3).

(a) Explain qualitatively what the music would sound like when played back, if the incorrectly encoded sampling frequency f_s' is given by,

$$f_s' = (3f_s)/4. \tag{7}$$

(b) Repeat part (a) for $f_s' = (4f_s)/3$.

Problem 5:

Compute $\arg(z)$ for the following complex numbers:

(a) $z = 1 + \mathbf{j}$

(b) $z = 1 - \mathbf{j}$

(c) $-1 - \mathbf{j}$

(d) $-1 + \mathbf{j}$

Problem 6:

Let $z_1 = (a + \mathbf{j}b)^2 e^{\mathbf{j}c}$, where a, b and c are real numbers. Solve the following expressions in terms of a, b and c. Your answers should not include the imaginary number $\mathbf{j}$.

(a) $Re[z_1]$

(b) $Im[z_1]$

(c) $Im[\mathbf{j}z_1]$

(d) $|z_1|$

(e) $\arg(z_1)$

(f) $Re[e^{\mathbf{j}\pi/4} z_1]$

Problem 7:

Let,

$$z_1 = -2 + \mathbf{j}2\sqrt{3} \text{ and } z_2 = 5e^{-\mathbf{j}\pi/3}$$

Solve the following expressions, giving your answers in both polar and rectangular form. You may verify your answers by computer, but must show all work required to arrive at the answers by hand calculation. Your answers should be exact (no numerical approximations).

(a) $2z_1 + z_2$

(b) $(z_1 z_2)^2$

(c) $z_1 z_1^*$

(d) z_1/z_2

(e) e^{z_2}

(f) $\sqrt{z_1}$

Problem 8:

Using the inverse Euler formulas, prove the following trigonometric identities:

(a) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

(b) $\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$

Problem 9:

Let $f(z) = \cos(\sqrt{z})$. Plot $|f(e^{\mathbf{j}\theta})|$ and $\arg[f(e^{\mathbf{j}\theta})]$ for $-\pi \le \theta < \pi$.