EEL3135: Homework #3

(14 problems, distributed 3/20/2002, due 3/28/2002)

Instructions:

Show/explain all work to get full credit.

Problem 1:

(a) Compute the CTFT for the following signal: $x(t) = \cos(2\pi t)\cos(3t) + \cos(4\pi t)$.

(b) Compute $x(t)$ for the following CTFT: $X(f) = 2e^{\mathbf{j}\pi/3}\delta(f+4) + 2e^{-\mathbf{j}\pi/3}\delta(f-4)$. Your final answer should not include the imaginary number $\mathbf{j}$.

Problem 2:

Consider the continuous-time signal:

$$x(t) = 5 + \cos(2\pi t) + 3\sin(7\pi t) + 10\cos(8\pi t - \pi/4),\ -\infty < t < \infty. \quad (2\text{-}1)$$

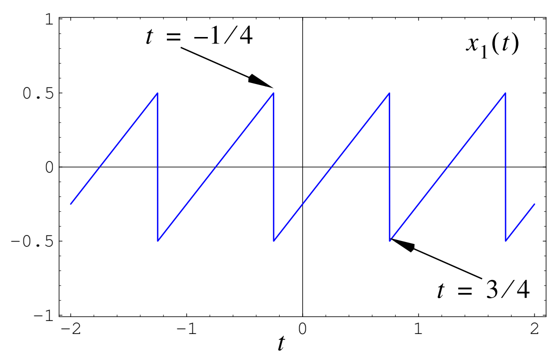
(a) What is the fundamental frequency f_0 of this periodic signal?

(b) Specify the Fourier series coefficients X_k, $-\infty < k < \infty$, for $x(t)$.

(c) Specify the Fourier series coefficients X'_k for the signal $x'(t) = 2x(t+1)$.

Problem 3:

Consider the periodic sawtooth waveform $x_1(t)$ plotted below.



(a) Derive the complex Fourier series coefficients X_k for this waveform.

<u>Hints</u>: Note that this is just a shifted version of the sawtooth wave $x(t)$ in Figure 4 (page 5, *Fourier Series* notes); that is,

$$x_1(t) = x(t - 1/4). \quad (3\text{-}1)$$

Therefore, you do not have to derive the X_k coefficients from scratch, although you are free to do so (not recommended though).

(b) Plot the frequency spectrum for this waveform. That is, plot $|X_k|$ and $\angle X_k$ as a function of k, $-50 \le k \le 50$.

(c) Plot the first 5 terms of the trigonometric Fourier series for $-2 \le t \le 2$.

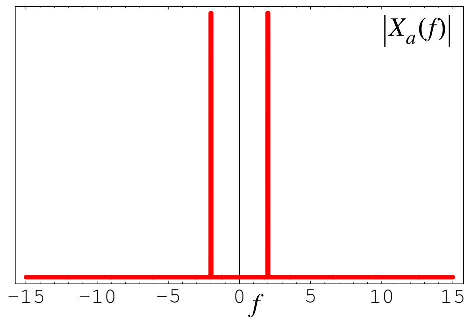
Problem 4:

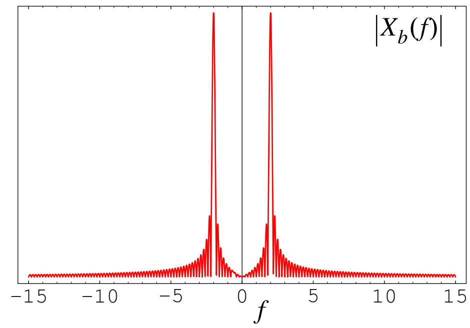
Below, there are two magnitude frequency spectra plotted, corresponding to the following two time-domain signals:

$$x_1(t) = \cos(4\pi t), \; -\infty < t < \infty \tag{4-1}$$

$$x_2(t) = \cos(4\pi t)[u(t) - u(t-5)], \; -\infty < t < \infty \tag{4-2}$$

Specify and explain which signal corresponds to which magnitude frequency spectrum.





Problem 5:

Below, the magnitudes of the DTFT are plotted for two sampled signals $x_1[n]$ and $x_2[n]$ given by,

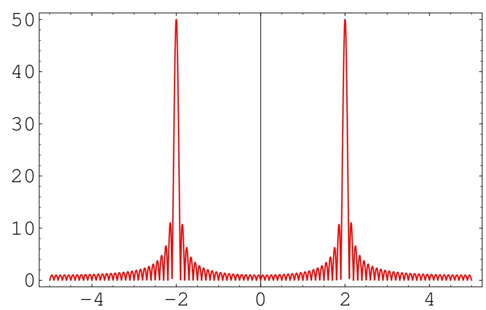
$$x_1[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 98, 99\} \\ 0 & \textit{elsewhere} \end{cases} \quad \text{where } x_c(t) = \cos(4\pi t) \text{, and} \tag{5-1}$$

$$x_2[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 48, 49\} \\ 0 & \textit{elsewhere} \end{cases} \quad \text{where } x_c(t) = \cos(4\pi t). \tag{5-2}$$

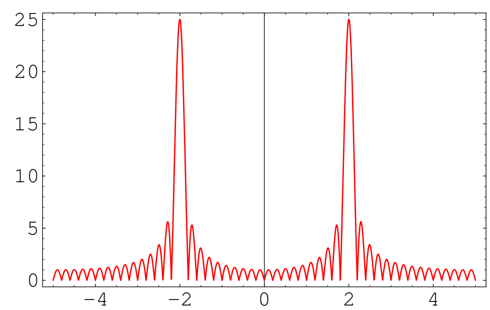
(a) Are the DTFTs plotted as a function of the frequency variable θ or frequency f (in Hz)?

(b) Specify and explain which discrete-time signal corresponds to which magnitude frequency spectrum.

Magnitude spectrum A (DTFT)



Magnitude spectrum B (DTFT)



Problem 6:

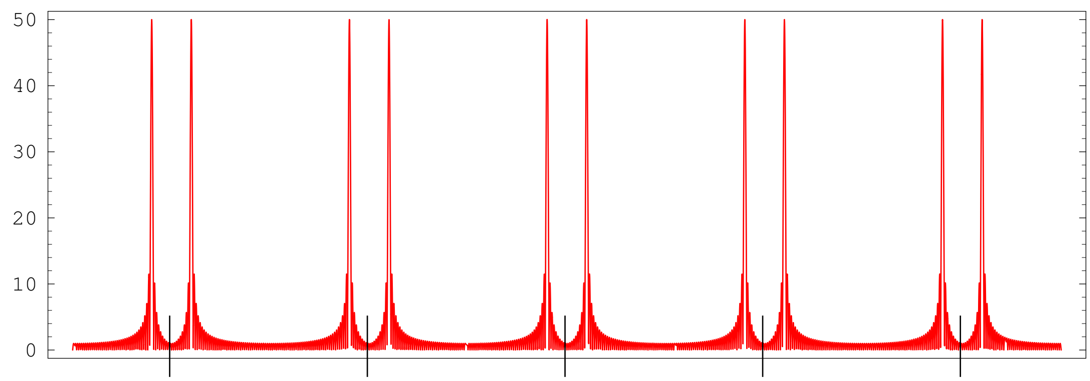
Below, the magnitude of the DTFT is plotted for the sampled signal $x[n]$ given by,

$$x[n] = \begin{cases} x_c(n/20) & n \in \{0, 1, \ldots, 98, 99\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = \cos(4\pi t). \tag{6-1}$$

(a) Label the horizontal axis where indicated, assuming the DTFT is plotted as a function of θ.

(b) Label the horizontal axis where indicated, assuming the DTFT is plotted as a function of f (in Hz).

Magnitude spectrum (DTFT)



Problem 7:

Below, two magnitude frequency spectra are plotted for two signals $x_1(t)$ and $x_2[n]$ given by,

$$x_1(t) = \cos(4\pi t)[u(t) - u(t-5)] \text{, and,} \tag{7-1}$$

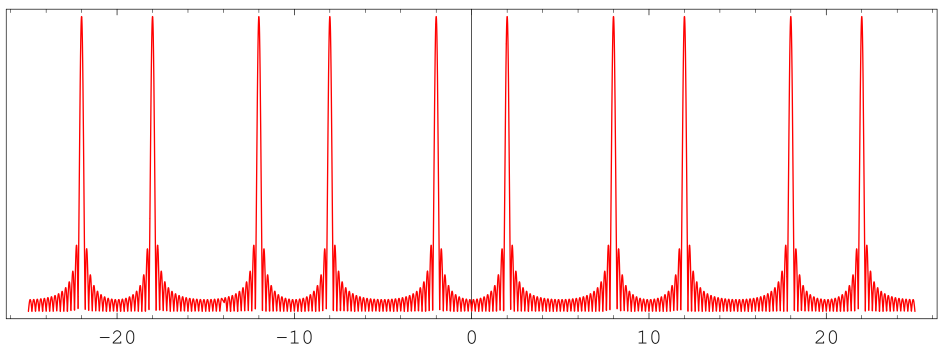
$$x_2[n] = \begin{cases} x_1(n/10) & n \in \{0, 1, \ldots, 48, 49\} \\ 0 & elsewhere \end{cases} \tag{7-2}$$

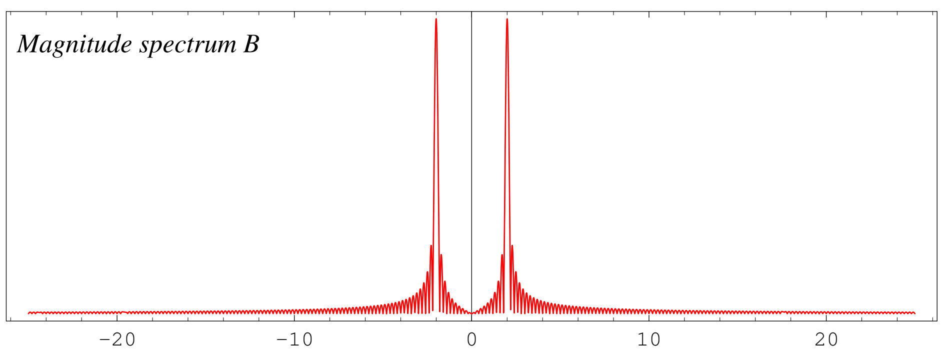
(a) To what variable does the horizontal axis in each plot correspond?

(b) Specify and explain which signal corresponds to which magnitude frequency spectrum.

(c) Specify to which frequency transform each magnitude spectrum corresponds (e.g. CTFT, DTFT, DFT).

Magnitude spectrum A





Problem 8:

Below, the magnitudes of the DTFT are plotted for two sampled signals $x_1[n]$ and $x_2[n]$ given by,

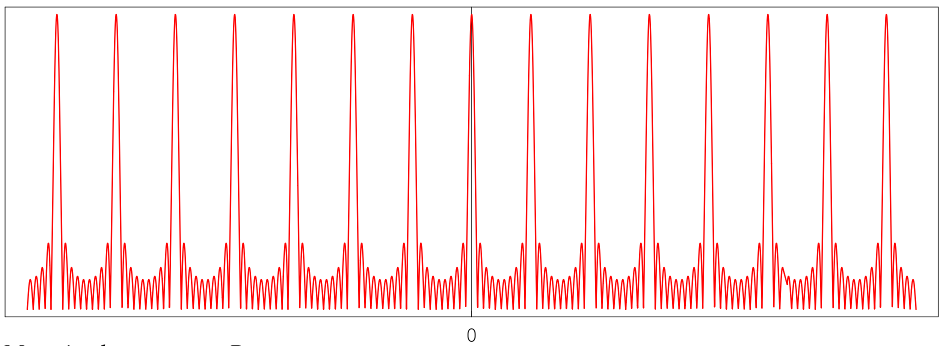
$$x_1[n] = \begin{cases} x_c(n/3) & n \in \{0, 1, \ldots, 28, 29\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = 1 + 2\cos(4\pi t)\text{, and} \tag{8-1}$$

$$x_2[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 28, 29\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = 1 + 2\cos(4\pi t)\text{.} \tag{8-2}$$

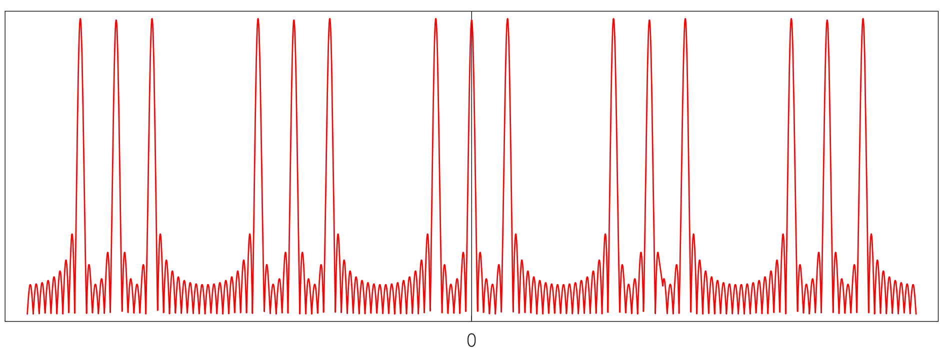
(a) Specify and explain which discrete-time signal corresponds to which magnitude frequency spectrum.

(b) Indicate to which frequency (in Hz) each of the dominant peaks in the plots corresponds.

Magnitude spectrum A



Magnitude spectrum B



Problem 9:

On the next page, the shape of ten magnitude frequency spectra are plotted as a function of frequency f (in Hertz) corresponding to ten of the following eleven time-domain signals:

$$x_1(t) = 1 + \sin(6\pi t), -\infty < t < \infty \quad (9\text{-}1)$$

$$x_2(t) = x_1(t)[u(t) - u(t-2)], -\infty < t < \infty \quad (9\text{-}2)$$

$$x_3(t) = x_1(t)[u(t) - u(t-3)], -\infty < t < \infty \quad (9\text{-}3)$$

$$x_4[n] = x_1(n/10), -\infty < n < \infty \quad (9\text{-}4)$$

$$x_5[n] = x_1(n/10)(u[n] - u[n-20]), -\infty < n < \infty \quad (9\text{-}5)$$

$$x_6[n] = x_1(n/20)(u[n] - u[n-20]), -\infty < n < \infty \quad (9\text{-}6)$$

$$x_7[n] = x_1(n/10)(u[n] - u[n-40]), -\infty < n < \infty \quad (9\text{-}7)$$

$$x_8[n] = x_1(n/6)(u[n] - u[n-24]), -\infty < n < \infty \quad (9\text{-}8)$$

$$x_9[n] = x_1(n/5)(u[n] - u[n-20]), -\infty < n < \infty \quad (9\text{-}9)$$

$$x_{10}[n] = x_1(n/4)(u[n] - u[n-16]), -\infty < n < \infty \quad (9\text{-}10)$$

$$x_{11}[n] = x_1(n/3)(u[n] - u[n-12]), -\infty < n < \infty \quad (9\text{-}11)$$

(Note that the first three signals are continuous-time signals, while the last eight signals are discrete-time signals, and that one of the above signals does not correspond to any of the magnitude spectra shown.)

(a) For each of the above time-domain signals, assign its corresponding frequency spectrum (i.e. *A, B, C, D, E, F, G, H, I, J*), or indicate "none" if none of the frequency spectra correspond to a particular signal.

(b) For each frequency spectrum, label it as either a CTFT or a DTFT.

Problem 10:

(a) Compute the DTFT $X(e^{j\theta})$ of the following discrete-time signal:

$$x[n] = \delta[n - n_0] \quad (10\text{-}1)$$

(b) For $n_0 = 2$, sketch $|X(e^{j\theta})|$, $-\pi < \theta < \pi$.

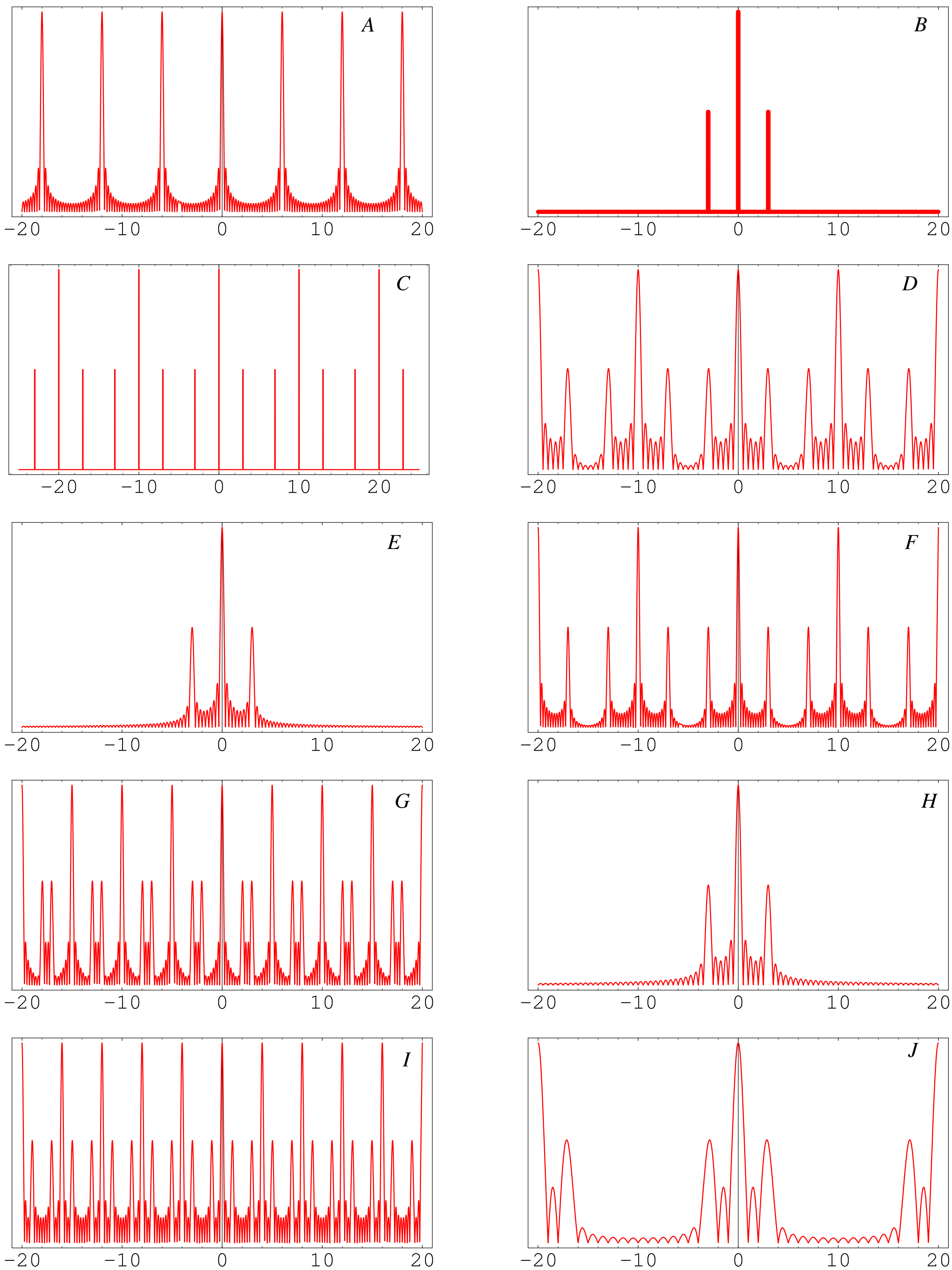
(c) For $n_0 = 2$, sketch $\angle X(e^{j\theta})$, $-\pi < \theta < \pi$.

(d) Compute the DTFT $X(e^{j\theta})$ of the following discrete-time signal:

$$x[n] = \delta[n-3] + \delta[n+3] \quad (10\text{-}2)$$

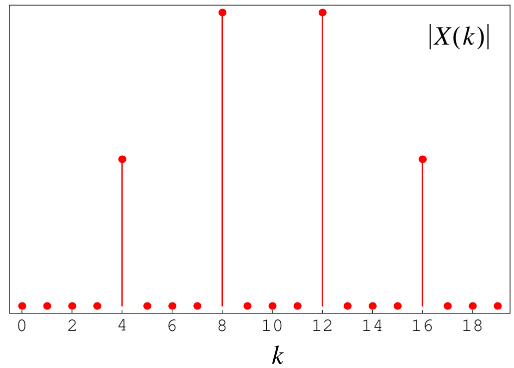
and plot $X(e^{j\theta})$.

Problem 9 figures *Magnitude frequency spectra (as function of frequency f)*



Problem 11:

(a) Assume the magnitude plot $|X(k)|$ of the DFT of a real-valued, discrete-time signal $x[n]$ is given by the plot below, and that $x[n]$ was sampled from a continuous-time signal at a sampling frequency of 10Hz. Specify the set of discrete-time signals consistent with $|X(k)|$.



(b) Suppose that $\angle X(16) = \pi/4$ and $\angle X(8) = -\pi/3$. Plot $\angle X(k)$ for $k \in \{0, 1, \ldots, 18, 19\}$.

(c) Sketch the magnitude DFT as a function of frequency (in Hertz).

(d) Which of the following frequencies cannot be represented exactly by this DFT?

$$f = 0.5 \text{ Hz}, f = 1.75 \text{ Hz}, f = 6 \text{ Hz}. \tag{11-1}$$

Problem 12:

Below, the magnitudes of the DFT are plotted for two sampled signals $x_1[n]$ and $x_2[n]$ given by,

$$x_1[n] = \begin{cases} x_{c1}(n/7) & n \in \{0, 1, \ldots, 18, 19\} \\ 0 & elsewhere \end{cases} \text{ where } x_{c1}(t) = \cos(2\pi t) \text{, and} \tag{12-1}$$

$$x_2[n] = \begin{cases} x_{c2}(n/7) & n \in \{0, 1, \ldots, 18, 19\} \\ 0 & elsewhere \end{cases} \text{ where } x_{c2}(t) = \cos(2\pi(21/20)t). \tag{12-2}$$

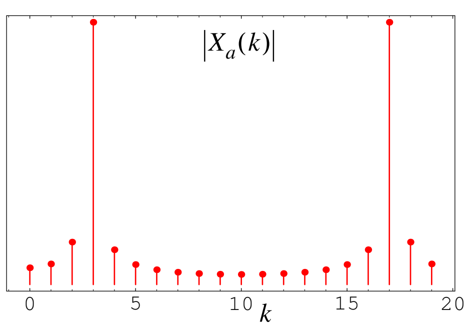
(a) Specify and explain which discrete-time signal corresponds to which magnitude frequency spectrum (DFT).

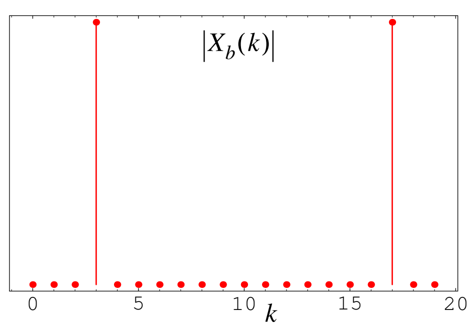
(b) For each of the indexes k below, indicate the corresponding frequency f:

$$k = 0, k = 3, k = 17. \tag{12-3}$$

(c) For each of the frequencies f below, indicate whether or not the DFT can represent those frequencies exactly:

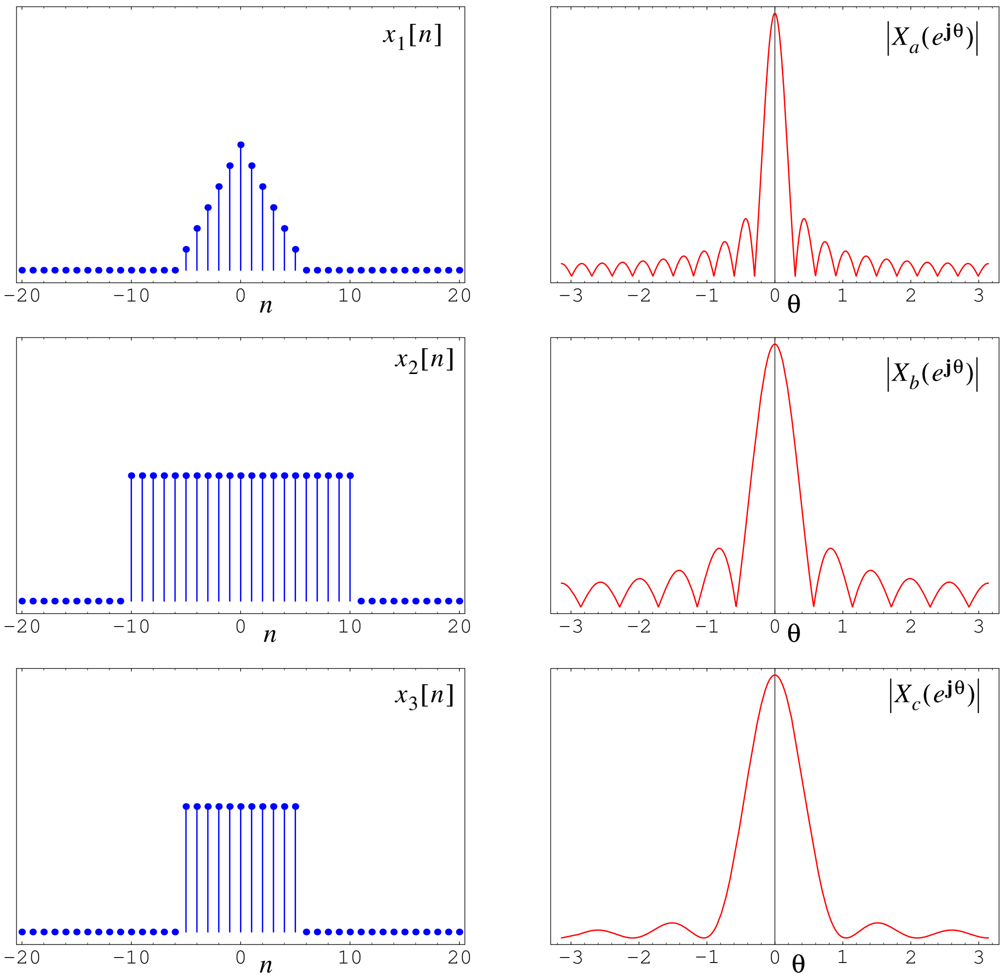
$$f = 0.35 \text{ Hz}, f = 1 \text{ Hz}, f = 1.4 \text{ Hz}, f = 1.5 \text{ Hz}. \tag{12-4}$$





Problem 13:

(a) Match each discrete-time signal to its corresponding magnitude DTFT representation. Explain your answer.



(b) Match each of the following discrete-time signals,

$$x_1[n] = x(n/3)(u[n] - u[n-15])\,,\ -\infty < n < \infty \tag{13-1}$$

$$x_2[n] = x(n/3)(u[n] - u[n-30])\,,\ -\infty < n < \infty \tag{13-2}$$

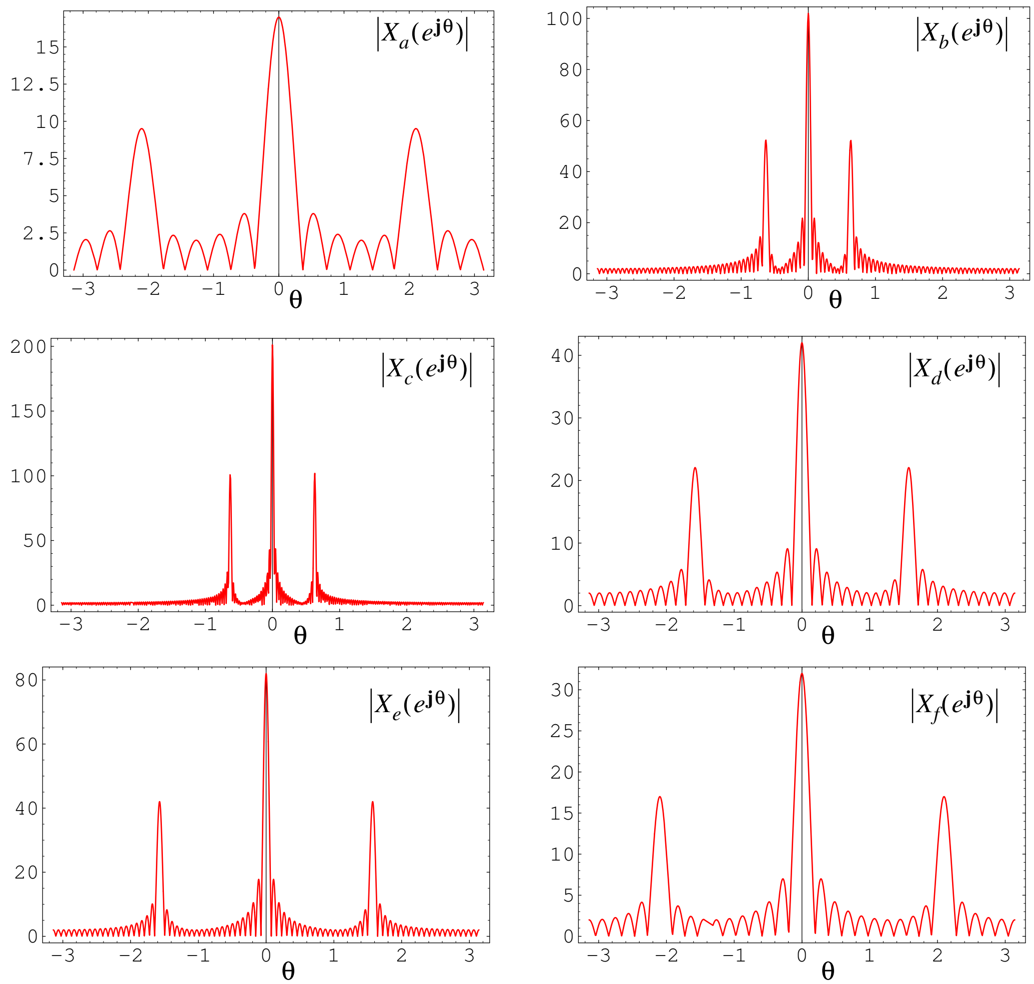
$$x_3[n] = x(n/8)(u[n] - u[n-40])\,,\ -\infty < n < \infty \tag{13-3}$$

$$x_4[n] = x(n/8)(u[n] - u[n-80])\,,\ -\infty < n < \infty \tag{13-4}$$

$$x_5[n] = x(n/20)(u[n] - u[n-100])\,,\ -\infty < n < \infty \tag{13-5}$$

$$x_6[n] = x(n/20)(u[n] - u[n-200])\,,\ -\infty < n < \infty \tag{13-6}$$

where $x(t) = 1 + \cos(4\pi t)$, $-\infty < t < \infty$, to its corresponding magnitude DTFT representation below. Recall that $\theta = (2\pi f)/f_s$. Explain your answer.



Problem 14:

(a) Write a computer program (MATLAB, Mathematica, etc.) to plot the magnitude and phase (as a function of θ) of the DTFT for the following discrete-time signal:

$$x_1[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 20\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = \cos(2\pi t) + 3\cos(4\pi t)\,. \qquad (14\text{-}1)$$

(a) Now plot the magnitude and phase (as a function of θ) of the DTFT for the following discrete-time signal:

$$x_2[n] = x_1[n+10] \qquad (14\text{-}2)$$

where $x_1[n]$ is given in (14-1) above.