EEL3135: Homework #3 Solutions

Problem 1:

(a) Compute the CTFT for the following signal: $x(t) = \cos(2\pi t)\cos(3t) + \cos(4\pi t)$.

First, we use the trigonometric identity (easy to show by using the inverse Euler relations):

$$\cos(2\pi t)\cos(3t) = \frac{1}{2}\cos[(3-2\pi)t] + \frac{1}{2}\cos[(3+2\pi)t] \quad \text{(S-1)}$$

$$\cos(2\pi t)\cos(3t) = \frac{1}{2}\cos[(2\pi-3)t] + \frac{1}{2}\cos[(3+2\pi)t] \quad \text{(S-2)}$$

Therefore, the CTFT is given by:

$$X(f) = \frac{1}{4}\left[\delta\left(f+\frac{(2\pi-3)}{2\pi}\right)+\delta\left(f-\frac{(2\pi-3)}{2\pi}\right)\right]+\frac{1}{4}\left[\delta\left(f+\frac{(3+2\pi)}{2\pi}\right)+\delta\left(f-\frac{(3+2\pi)}{2\pi}\right)\right]+ \frac{1}{2}[\delta(f+2)+\delta(f-2)] \quad \text{(S-3)}$$

(b) Compute $x(t)$ for the following CTFT: $X(f) = 2e^{\mathbf{j}\pi/3}\delta(f+4) + 2e^{-\mathbf{j}\pi/3}\delta(f-4)$. Your final answer should not include the imaginary number $\mathbf{j}$.

From the definition of the inverse Fourier transform:

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{\mathbf{j}2\pi ft}df \quad \text{(S-4)}$$

$$x(t) = \int_{-\infty}^{\infty} [2e^{\mathbf{j}\pi/3}\delta(f+4) + 2e^{-\mathbf{j}\pi/3}\delta(f-4)]e^{\mathbf{j}2\pi ft}df \quad \text{(S-5)}$$

$$x(t) = 2e^{\mathbf{j}\pi/3}e^{-\mathbf{j}8\pi t} + 2e^{-\mathbf{j}\pi/3}e^{\mathbf{j}8\pi t} \quad \text{(S-6)}$$

$$x(t) = 2[e^{-\mathbf{j}(8\pi t-\pi/3)} + e^{\mathbf{j}(8\pi t-\pi/3)}] \quad \text{(S-7)}$$

$$x(t) = 4\left[\frac{e^{-\mathbf{j}(8\pi t-\pi/3)} + e^{\mathbf{j}(8\pi t-\pi/3)}}{2}\right] \quad \text{(S-8)}$$

$$x(t) = 4\cos(8\pi t - \pi/3) \quad \text{(S-9)}$$

Problem 2:

Consider the continuous-time signal:

$$x(t) = 5 + \cos(2\pi t) + 3\sin(7\pi t) + 10\cos(8\pi t - \pi/4),\ -\infty < t < \infty. \tag{2-1}$$

(a) What is the fundamental frequency f_0 of this periodic signal?

Note that $x(t)$ is already in the form of a Fourier series:

$$x(t) = X_0 + 2\sum_{k=1}^{\infty} |X_k| \cos(2\pi k f_0 t + \angle X_k) \tag{S-10}$$

So, we do not need to compute the Fourier series coefficients through integration. Now, in the Fourier series, all the frequencies have to be integer multiples of $2\pi f_0$. By inspection, $f_0 = 1/2$ Hz, so that the four terms in equation (2-1) correspond to $k = 0$, $k = \pm 2$, $k = \pm 7$ and $k = \pm 8$.

(b) Specify the Fourier series coefficients X_k, $-\infty < k < \infty$, for $x(t)$.

By comparing equation (2-1) and (S-10), we can find X_k by inspection:

$$X_0 = 5 \tag{S-11}$$

$$X_2 = 1/2 \tag{S-12}$$

$$X_7 = 3/2e^{-\mathbf{j}\pi/2} = -3\mathbf{j}/2\ [\sin(7\pi t) = \cos(7\pi t - \pi/2)] \tag{S-13}$$

$$X_8 = 10/2e^{-\mathbf{j}\pi/4} = 5e^{-\mathbf{j}\pi/4} \tag{S-14}$$

Using the property that $X_{-k} = X_k^*$:

$$X_{-2} = 1/2 \tag{S-15}$$

$$X_{-7} = 3\mathbf{j}/2 \tag{S-16}$$

$$X_{-8} = 5e^{\mathbf{j}\pi/4} \tag{S-17}$$

All other Fourier series coefficients are zero.

(c) Specify the Fourier series coefficients X'_k for the signal $x'(t) = 2x(t+1)$.

Starting with the complex Fourier series representation:

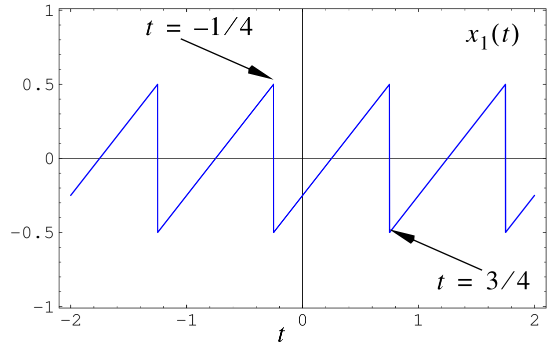
$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{\mathbf{j}\pi k t} \tag{S-18}$$

$$2x(t+1) = \sum_{k=-\infty}^{\infty} 2X_k e^{\mathbf{j}\pi k(t+1)} = \sum_{k=-\infty}^{\infty} (2X_k e^{\mathbf{j}\pi k}) e^{\mathbf{j}\pi k t} \tag{S-19}$$

Therefore $X_k' = 2X_k e^{\mathbf{j}\pi k}$.

Problem 3:

Consider the periodic sawtooth waveform $x_1(t)$ plotted below.



(a) Derive the complex Fourier series coefficients X_k for this waveform.

<u>Hints</u>: Note that this is just a shifted version of the sawtooth wave $x(t)$ in Figure 4 (page 5, *Fourier Series* notes); that is,

$$x_1(t) = x(t - 1/4)\,. \tag{3-1}$$

Therefore, you do not have to derive the X_k coefficients from scratch, although you are free to do so (not recommended though).

In the *Fourier Series* lecture notes, we derived the following Fourier series coefficients for the unshifted sawtooth waveform $x(t)$:

$$X_k = \begin{cases} (\mathbf{j}(-1)^k)/(2\pi k) & k \neq 0 \\ 0 & k = 0 \end{cases} \tag{S-20}$$

where the complex Fourier series representation is given by,

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{\mathbf{j}2\pi k t} \tag{S-21}$$

Substituting $(t - 1/4)$ for t in equation (S-21):

$$x(t - 1/4) = \sum_{k=-\infty}^{\infty} X_k e^{\mathbf{j}2\pi k(t - 1/4)} \tag{S-22}$$

$$x(t - 1/4) = \sum_{k=-\infty}^{\infty} X_k e^{\mathbf{j}2\pi k t} e^{\mathbf{j}2\pi k(-1/4)} \tag{S-23}$$

$$x(t - 1/4) = \sum_{k=-\infty}^{\infty} (X_k e^{-\mathbf{j}\pi k/2}) e^{\mathbf{j}2\pi k t} \tag{S-24}$$

From equation (S-24), we observe that the Fourier coefficients X_k' for the shifted waveform are given by,

$$X_k' = X_k e^{-\mathbf{j}\pi k/2} \tag{S-25}$$

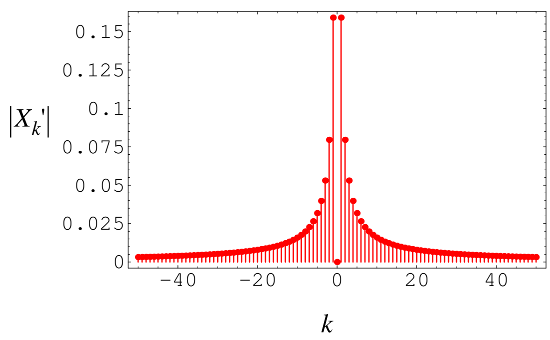
$$X_k' = \begin{cases} (\mathbf{j}(-1)^k e^{-\mathbf{j}\pi k/2})/(2\pi k) & k \neq 0 \\ 0 & k = 0 \end{cases} \tag{S-26}$$

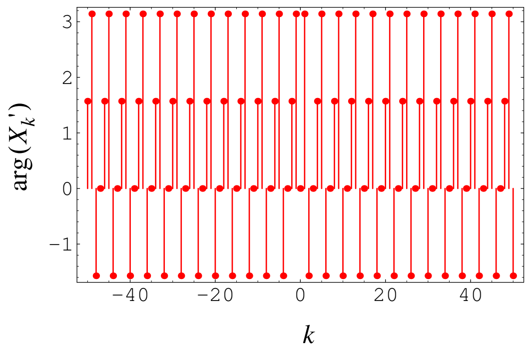
Equation (S-26) can be simplified to:

$$X_k' = \begin{cases} (\mathbf{j}^{(k+1)})/(2\pi k) & k \neq 0 \\ 0 & k = 0 \end{cases} \qquad \text{(S-27)}$$

(The result in (S-27) can be verified by direct substitution of integers k.)

(b) Plot the frequency spectrum for this waveform. That is, plot $|X_k|$ and $\angle X_k$ as a function of k, $-50 \le k \le 50$.





(c) Plot the first 5 terms of the trigonometric Fourier series for $-2 \le t \le 2$.

The trigonometric representation of the Fourier series for the shifted waveform is given by,

$$x(t - 1/4) = X_0' + 2\sum_{k=1}^{\infty} |X_k'| \cos(2\pi k t + \angle X_k') \qquad \text{(S-28)}$$

So the fifth-order approximation is given by,

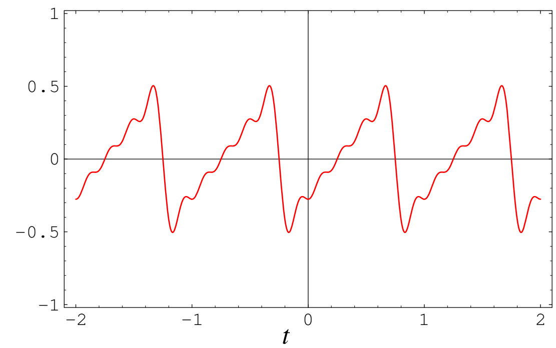
$$x(t - 1/4) \approx 2\sum_{k=1}^{5} |X_k'| \cos(2\pi k t + \angle X_k') \qquad \text{(S-29)}$$

$$x(t - 1/4) \approx \frac{\cos(2\pi t + \pi)}{\pi} + \frac{\cos(4\pi t - \pi/2)}{2\pi} + \frac{\cos(6\pi t)}{3\pi} + \frac{\cos(8\pi t + \pi/2)}{4\pi} + \frac{\cos(10\pi t + \pi)}{5\pi} \qquad \text{(S-30)}$$

$$x(t - 1/4) \approx \frac{-\cos(2\pi t)}{\pi} + \frac{\sin(4\pi t)}{2\pi} + \frac{\cos(6\pi t)}{3\pi} - \frac{\sin(8\pi t)}{4\pi} - \frac{\cos(10\pi t)}{5\pi} \qquad \text{(S-31)}$$

Equation (S-31) is plotted below. (For MATLAB and Mathematica code, see the course web page.)

Truncated Fourier series



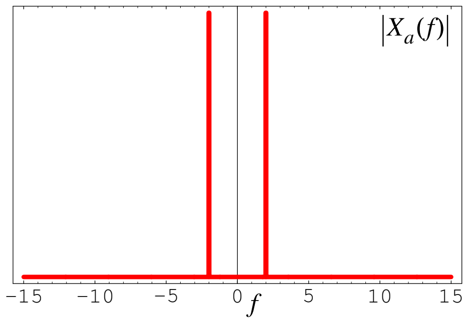
Problem 4:

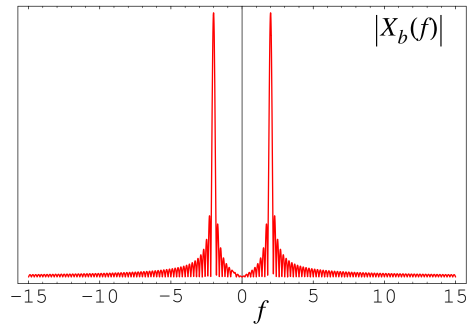
Below, there are two magnitude frequency spectra plotted, corresponding to the following two time-domain signals:

$$x_1(t) = \cos(4\pi t)\,,\ -\infty < t < \infty \tag{4-1}$$

$$x_2(t) = \cos(4\pi t)[u(t) - u(t-5)]\,,\ -\infty < t < \infty \tag{4-2}$$

Specify and explain which signal corresponds to which magnitude frequency spectrum.





The equations below give the correct correspondences between the time and frequency domain plots:

$$x_1(t) \Leftrightarrow |X_a(f)| \tag{S-32}$$

$$x_2(t) \Leftrightarrow |X_b(f)| \tag{S-33}$$

Note that $X_a(f)$ is given by,

$$X_a(f) = \frac{1}{2}\delta(f+2) + \frac{1}{2}\delta(f-2)\,. \tag{S-34}$$

Also, note that the time limiting of signal $x_2(t)$ causes the frequency content to spread out in the frequency domain.

Problem 5:

Below, the magnitudes of the DTFT are plotted for two sampled signals $x_1[n]$ and $x_2[n]$ given by,

$$x_1[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 98, 99\} \\ 0 & \mathit{elsewhere} \end{cases} \text{ where } x_c(t) = \cos(4\pi t) \text{, and} \quad (5\text{-}1)$$

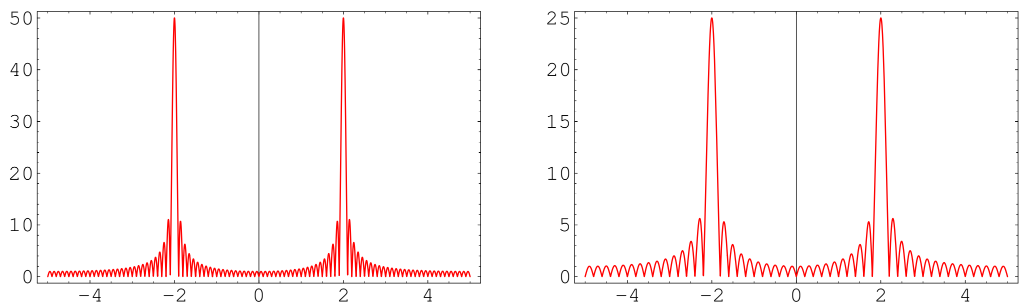
$$x_2[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 48, 49\} \\ 0 & \mathit{elsewhere} \end{cases} \text{ where } x_c(t) = \cos(4\pi t) \text{.} \quad (5\text{-}2)$$

(a) Are the DTFTs plotted as a function of the frequency variable θ or frequency f (in Hz)?

Since we should expect peaks in the DTFTs at ±2 Hz, we note that the DTFTs are plotted as a function of frequency f.

(b) Specify and explain which discrete-time signal corresponds to which magnitude frequency spectrum.

Magnitude spectrum A (DTFT) *Magnitude spectrum B (DTFT)*



The equations below give the correct correspondences between the time and frequency domain plots:

$$x_1(t) \Leftrightarrow \mathit{spectrum}\ A \quad (S\text{-}35)$$

$$x_2(t) \Leftrightarrow \mathit{spectrum}\ B \quad (S\text{-}36)$$

The signal $x_1[n]$ results from longer sampling than $x_2[n]$ and therefore its frequency content is more localized about the dominant frequencies of ±2 Hz.

Problem 6:

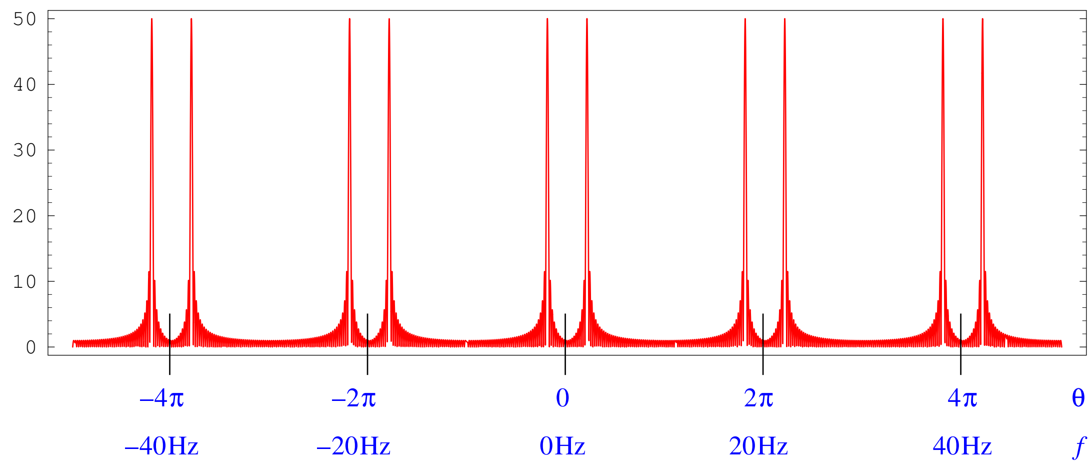
Below, the magnitude of the DTFT is plotted for the sampled signal $x[n]$ given by,

$$x[n] = \begin{cases} x_c(n/20) & n \in \{0, 1, \ldots, 98, 99\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = \cos(4\pi t). \tag{6-1}$$

(a) Label the horizontal axis where indicated, assuming the DTFT is plotted as a function of θ.

(b) Label the horizontal axis where indicated, assuming the DTFT is plotted as a function of f (in Hz).

Magnitude spectrum (DTFT)



Note the correct labeling of the horizontal axis as a function of both θ and f. Remember that the DTFT is periodic with period 2π, and that, as a function of frequency, the spectrum of the continuous-time waveform is replicated at $\pm k f_s$, where k is an integer.

Problem 7:

Below, two magnitude frequency spectra are plotted for two signals $x_1(t)$ and $x_2[n]$ given by,

$$x_1(t) = \cos(4\pi t)[u(t) - u(t-5)] \text{, and,} \tag{7-1}$$

$$x_2[n] = \begin{cases} x_1(n/10) & n \in \{0, 1, \ldots, 48, 49\} \\ 0 & elsewhere \end{cases} \tag{7-2}$$

(a) To what variable does the horizontal axis in each plot correspond?

The magnitude spectra are plotted as a function of frequency f in Hertz.

(b) Specify and explain which signal corresponds to which magnitude frequency spectrum.

The equations below give the correct correspondences between the time and frequency domain plots:

$$x_1(t) \Leftrightarrow spectrum\ B \tag{S-37}$$

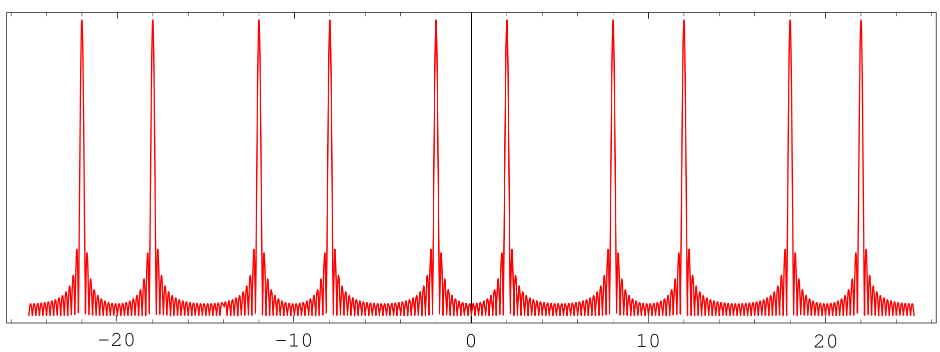
$$x_2[n] \Leftrightarrow spectrum\ A \tag{S-38}$$

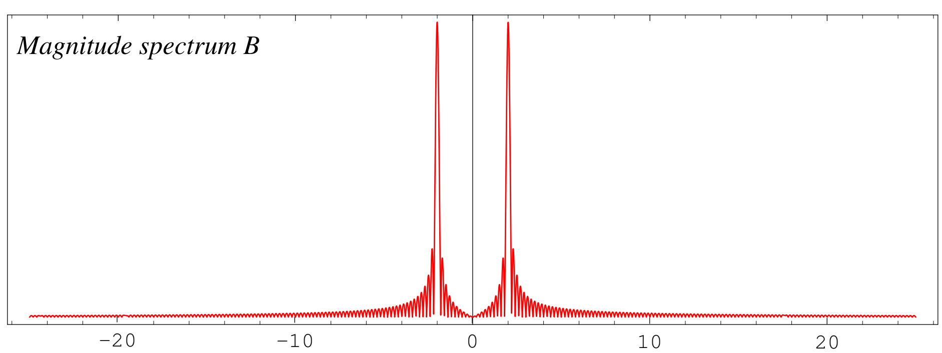
Note that the time-limited continuous-time signal $x_1(t)$ has dominant frequencies at $\pm 2\text{Hz}$, while the sampled version of the signal has the original spectrum replicated at $\pm k f_s$, where k is an integer.

(c) Specify to which frequency transform each magnitude spectrum corresponds (e.g. CTFT, DTFT, DFT).

Magnitude spectrum A corresponds to the DTFT, while *magnitude spectrum B* corresponds to the CTFT.

Magnitude spectrum A





Problem 8:

Below, the magnitudes of the DTFT are plotted for two sampled signals $x_1[n]$ and $x_2[n]$ given by,

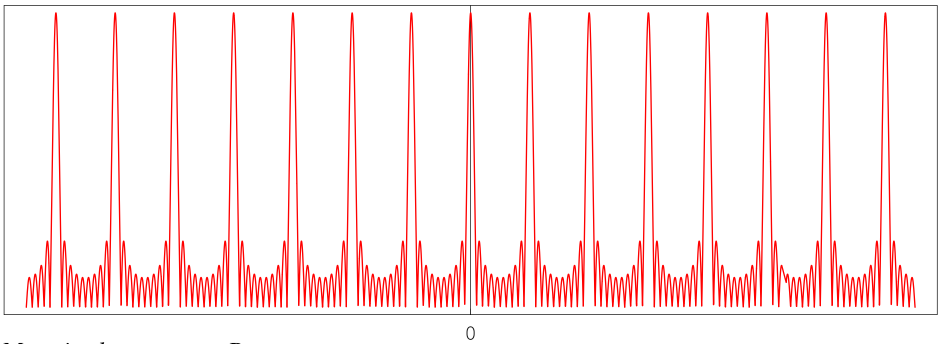
$$x_1[n] = \begin{cases} x_c(n/3) & n \in \{0, 1, \ldots, 28, 29\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = 1 + 2\cos(4\pi t)\text{, and} \tag{8-1}$$

$$x_2[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 28, 29\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = 1 + 2\cos(4\pi t)\text{.} \tag{8-2}$$

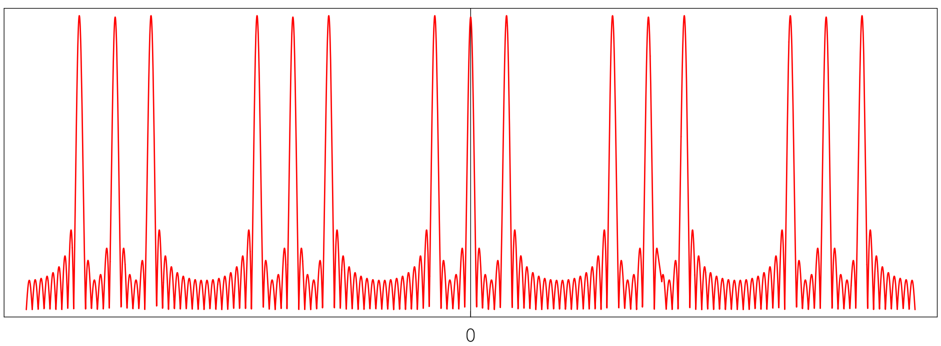
(a) Specify and explain which discrete-time signal corresponds to which magnitude frequency spectrum.

(b) Indicate to which frequency (in Hz) each of the dominant peaks in the plots corresponds.

Magnitude spectrum A



Magnitude spectrum B



The equations below give the correct correspondences between the time and frequency domain plots:

$$x_1[n] \Leftrightarrow spectrum\ A \tag{S-39}$$

$$x_2[n] \Leftrightarrow spectrum\ B \tag{S-40}$$

Note that the sampling frequency of 3Hz for $x_1[n]$ is below the Nyquist sampling frequency of 4Hz, and so aliasing occurs. The peaks for *spectrum A* correspond to $\pm k\text{Hz}$, where k is an integer. On the other had, the sampling frequency of 10Hz for $x_2[n]$ is above the Nyquist sampling frequency of 4Hz, and so aliasing does not occur. The peaks for *spectrum B* correspond to:

$$\{-2 + 10k, 0 + 10k, 2 + 10k\} \text{ (in Hertz)} \tag{S-41}$$

where k is an integer.

To understand these plots, you can perform a similar analysis as for problems 1 and 2 on homework #2, assuming infinite-length sampling of $x_c(t)$.

Problem 9:

On the next page, the shape of ten magnitude frequency spectra are plotted as a function of frequency f (in Hertz) corresponding to ten of the following eleven time-domain signals:

B/CTFT $$x_1(t) = 1 + \sin(6\pi t), -\infty < t < \infty \quad (9\text{-}1)$$

H/CTFT $$x_2(t) = x_1(t)[u(t) - u(t-2)], -\infty < t < \infty \quad (9\text{-}2)$$

E/CTFT $$x_3(t) = x_1(t)[u(t) - u(t-3)], -\infty < t < \infty \quad (9\text{-}3)$$

C/DTFT $$x_4[n] = x_1(n/10), -\infty < n < \infty \quad (9\text{-}4)$$

D/DTFT $$x_5[n] = x_1(n/10)(u[n] - u[n-20]), -\infty < n < \infty \quad (9\text{-}5)$$

J/DTFT $$x_6[n] = x_1(n/20)(u[n] - u[n-20]), -\infty < n < \infty \quad (9\text{-}6)$$

F/DTFT $$x_7[n] = x_1(n/10)(u[n] - u[n-40]), -\infty < n < \infty \quad (9\text{-}7)$$

A/DTFT $$x_8[n] = x_1(n/6)(u[n] - u[n-24]), -\infty < n < \infty \quad (9\text{-}8)$$

G/DTFT $$x_9[n] = x_1(n/5)(u[n] - u[n-20]), -\infty < n < \infty \quad (9\text{-}9)$$

I/DTFT $$x_{10}[n] = x_1(n/4)(u[n] - u[n-16]), -\infty < n < \infty \quad (9\text{-}10)$$

none $$x_{11}[n] = x_1(n/3)(u[n] - u[n-12]), -\infty < n < \infty \quad (9\text{-}11)$$

(Note that the first three signals are continuous-time signals, while the last eight signals are discrete-time signals, and that one of the above signals does not correspond to any of the magnitude spectra shown.)

(a) For each of the above time-domain signals, assign its corresponding frequency spectrum (i.e. *A, B, C, D, E, F, G, H, I, J*), or indicate "none" if none of the frequency spectra correspond to a particular signal.

See the above labeling. The reason the DTFT of signal $x_8[n]$ does not have multiple peaks is because of the discrete sequence generated by a sampling frequency of $f_s = 6$ Hz is given by:

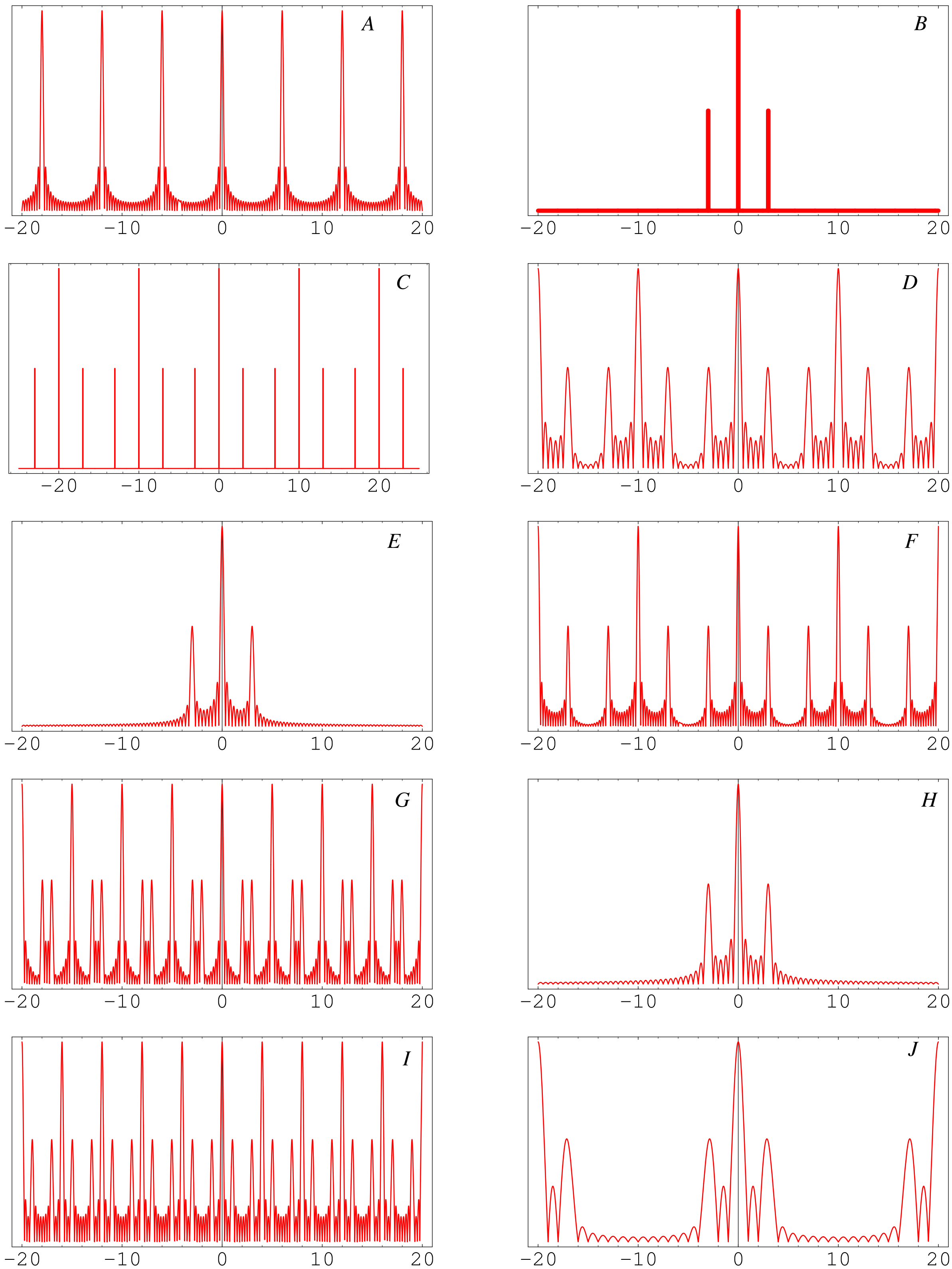
$$x_8[n] = 1 + \sin\left(\frac{6\pi n}{6}\right) = 1 + \sin(\pi n) = 1, \; n \in \{0, 1, \ldots, 22, 23\}. \quad (S\text{-}42)$$

That is, it represents a discrete time pulse of width 24. Also, note the aliasing for signals $x_9[n]$ and $x_{10}[n]$.

(b) For each frequency spectrum, label it as either a CTFT or a DTFT.

See the above labeling.

Problem 9 figures *Magnitude frequency spectra (as function of frequency f)*



Problem 10:

(a) Compute the DTFT $X(e^{j\theta})$ of the following discrete-time signal:

$$x[n] = \delta[n-n_0] \tag{10-1}$$

From the definition of the DTFT:

$$X(e^{j\theta}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\theta} \tag{S-43}$$

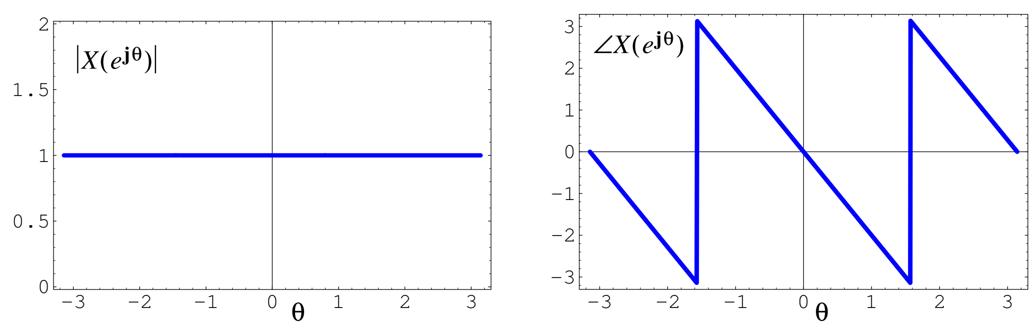
$$X(e^{j\theta}) = \delta[n-n_0]e^{-jn_0\theta} = e^{-jn_0\theta} \tag{S-44}$$

(b) For $n_0 = 2$, sketch $|X(e^{j\theta})|$, $-\pi < \theta < \pi$.

For $n_0 = 2$,

$$X(e^{j\theta}) = e^{-j2\theta} \tag{S-45}$$

See the magnitude plot below.



(c) For $n_0 = 2$, sketch $\angle X(e^{j\theta})$, $-\pi < \theta < \pi$.

See the phase plot above.

(d) Compute the DTFT $X(e^{j\theta})$ of the following discrete-time signal:

$$x[n] = \delta[n-3] + \delta[n+3] \tag{10-2}$$

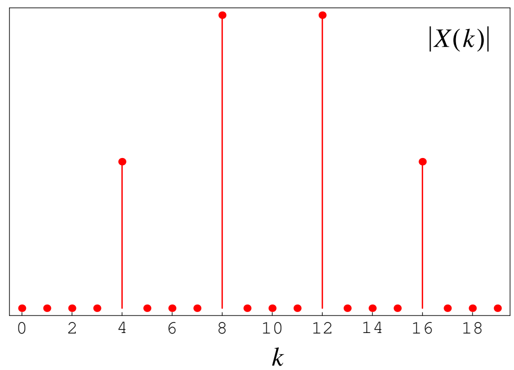
and plot $X(e^{j\theta})$.

From the definition of the DTFT:

$$X(e^{j\theta}) = \delta[n-3]e^{-j3\theta} + \delta[n+3]e^{j3\theta} = e^{-j3\theta} + e^{j3\theta} = 2\cos(3\theta) \tag{S-46}$$

Problem 11:

(a) Assume the magnitude plot $|X(k)|$ of the DFT of a real-valued, discrete-time signal $x[n]$ is given by the plot below, and that $x[n]$ was sampled from a continuous-time signal at a sampling frequency of 10Hz. Specify the set of discrete-time signals consistent with $|X(k)|$.



The length of the DFT is $N = 20$, and the sampling frequency is given as $f_s = 10$ Hz. Therefore,

$$k = 4 \quad \Leftrightarrow \quad f = 4(10/20) = 2\text{ Hz} \tag{S-47}$$

$$k = 16 \quad \Leftrightarrow \quad f = (16-20)(10/20) = -2\text{ Hz} \tag{S-48}$$

$$k = 8 \quad \Leftrightarrow \quad f = 8(10/20) = 4\text{ Hz} \tag{S-49}$$

$$k = 12 \quad \Leftrightarrow \quad f = (12-20)(10/20) = -4\text{ Hz} \tag{S-50}$$

Since we are not given the phase of the DFT or the absolute magnitude, the set of discrete-time signals $x[n]$ that are consistent with $|X(k)|$ is given by:

$$x[n] = A\left[\cos\left(\frac{4\pi n}{10} + \alpha\right) + 2\cos\left(\frac{8\pi n}{10} + \beta\right)\right] = A\left[\cos\left(\frac{2\pi n}{5} + \alpha\right) + 2\cos\left(\frac{4\pi n}{5} + \beta\right)\right] \tag{S-51}$$

where A, α and β can be any real-valued scalars.

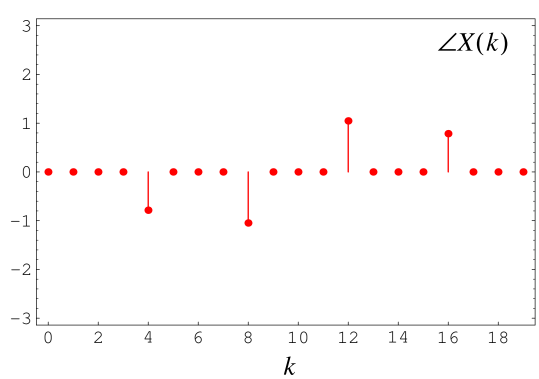
(b) Suppose that $\angle X(16) = \pi/4$ and $\angle X(8) = -\pi/3$. Plot $\angle X(k)$ for $k \in \{0, 1, \ldots, 18, 19\}$.

For all the zero DFT components $\angle X(k) = 0$. For the other two nonzero components:

$$\angle X(4) = -\angle X(16) = -\pi/4 \tag{S-52}$$

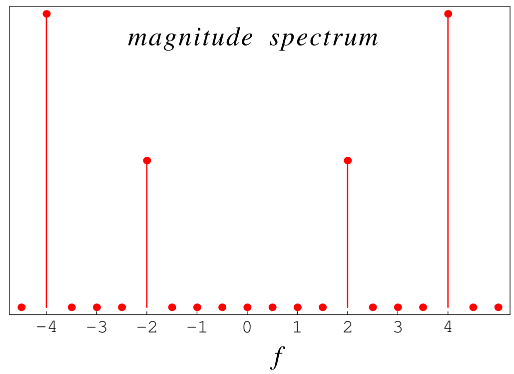
$$\angle X(12) = -\angle X(8) = \pi/3 \tag{S-53}$$

See the plot below for the phase DFT as a function of index k.



(c) Sketch the magnitude DFT as a function of frequency (in Hertz).

See the plot below for the magnitude DFT as a function of frequency f.



(d) Which of the following frequencies cannot be represented exactly by this DFT?

$$f = 0.5\text{ Hz}, f = 1.75\text{ Hz}, f = 6\text{ Hz}. \tag{11-1}$$

The frequency resolution of this DFT is $(10/20) = 0.5$ Hz. Therefore, $f = 1.75$ Hz cannot be represented by this DFT. Also, since the sampling frequency is 10Hz, the maximum frequency that can be represented by this DFT is $(10/2) = 5$ Hz. Therefore, $f = 6$ Hz cannot be represented by this DFT.

Problem 12:

Below, the magnitudes of the DFT are plotted for two sampled signals $x_1[n]$ and $x_2[n]$ given by,

$$x_1[n] = \begin{cases} x_{c1}(n/7) & n \in \{0, 1, \ldots, 18, 19\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_{c1}(t) = \cos(2\pi t)\text{, and} \tag{12-1}$$

$$x_2[n] = \begin{cases} x_{c2}(n/7) & n \in \{0, 1, \ldots, 18, 19\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_{c2}(t) = \cos(2\pi(21/20)t)\text{.} \tag{12-2}$$

(a) Specify and explain which discrete-time signal corresponds to which magnitude frequency spectrum (DFT).

The equations below give the correct correspondences between the time and frequency domain plots:

$$x_1[n] \Leftrightarrow |X_a(k)| \tag{S-54}$$

$$x_2[n] \Leftrightarrow |X_b(k)| \tag{S-55}$$

This is because the sampled signal $x_{c2}(t)$ has frequency 21/20Hz, which is an integer multiple of $f_s/N = 7/20$ Hz, while the sampled signal $x_{c1}(t)$ has frequency 1Hz, which is not an integer multiple of $7/20$ Hz.

(b) For each of the indexes k below, indicate the corresponding frequency f:

$$k = 0\text{, } k = 3\text{, } k = 17\text{.} \tag{12-3}$$

For the DFT, the frequency resolution is given by f_s/N, which for this example is given by $7/20$ Hz. Then,

$$\{k = 0\} \Leftrightarrow \{0\text{Hz}\} \tag{S-56}$$

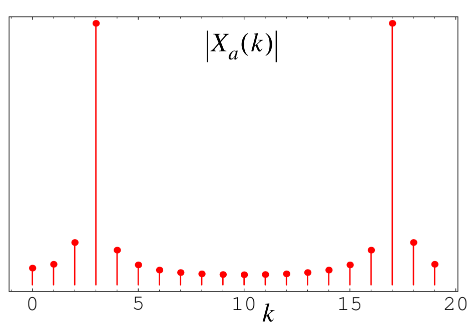
$$\{k = 3\} \Leftrightarrow \{3(7/20)\text{Hz} = 21/20\text{Hz}\} \tag{S-57}$$

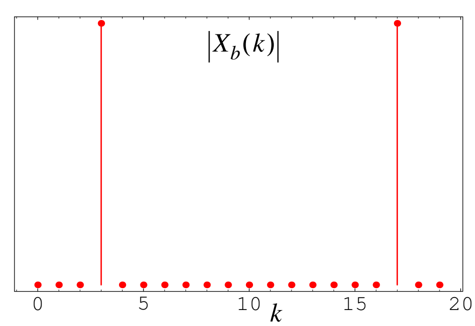
$$\{k = 17\} \Leftrightarrow \{3(17-20)/20\text{Hz} = -21/20\text{Hz}\} \tag{S-58}$$

(c) For each of the frequencies f below, indicate whether or not the DFT can represent those frequencies exactly:

$$f = 0.35\text{ Hz, } f = 1\text{ Hz, } f = 1.4\text{ Hz, } f = 1.5\text{ Hz.} \tag{12-4}$$

The DFT can only represent integer multiples of $7/20 = 0.35$ Hz exactly. Therefore, 0.35Hz and 1.4Hz can be represented exactly in the DFT representation.



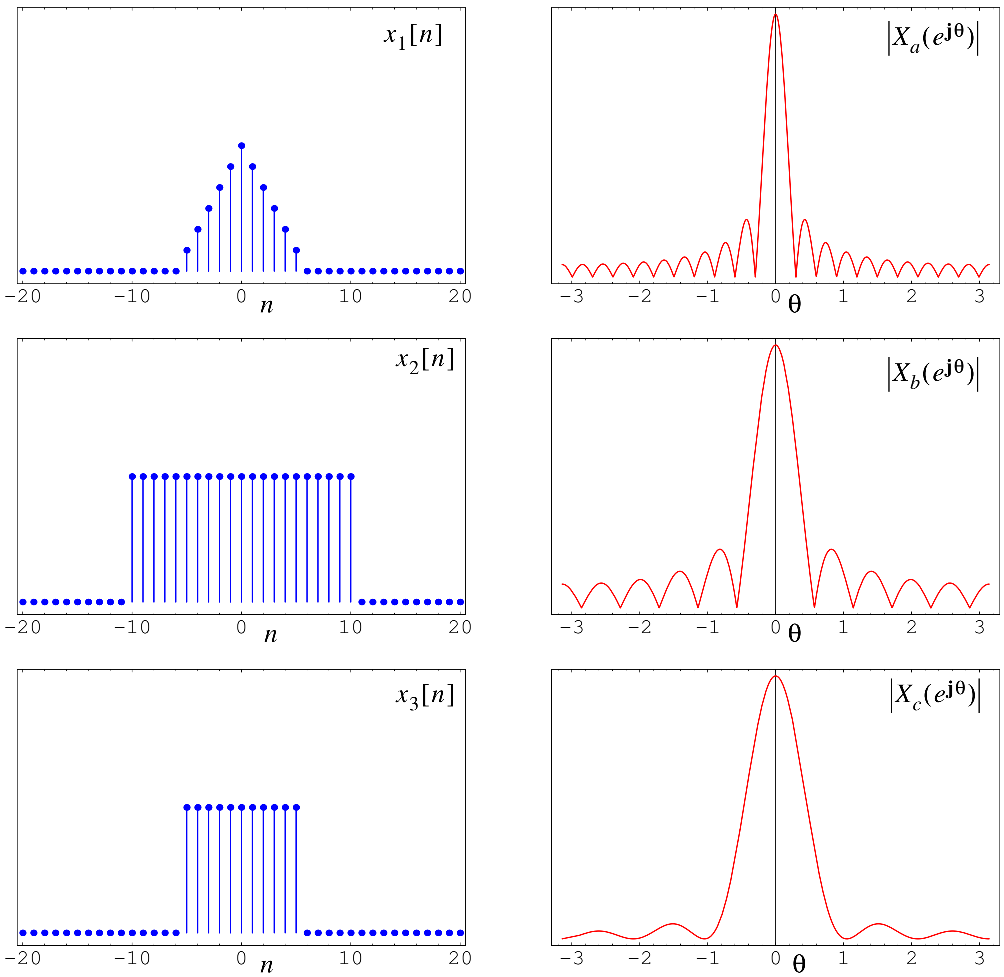


Problem 13:

(a) Match each discrete-time signal to its corresponding magnitude DTFT representation. Explain your answer.

$$x_1[n] \Leftrightarrow |X_c(e^{j\theta})|,\ x_2[n] \Leftrightarrow |X_a(e^{j\theta})|,\ x_3[n] \Leftrightarrow |X_b(e^{j\theta})| \quad \text{(S-59)}$$

Explanation: $x_1[n]$ is a windowed pulse function, and should therefore exhibit less spectral leakage than the other two signals; $x_2[n]$ is spread out more in the time domain than $x_3[n]$, and therefore will result in a more focused representation in the frequency domain.



(b) Match each of the following discrete-time signals,

$$x_1[n] = x(n/3)(u[n] - u[n-15]),\ -\infty < n < \infty \quad \text{(13-1)}$$

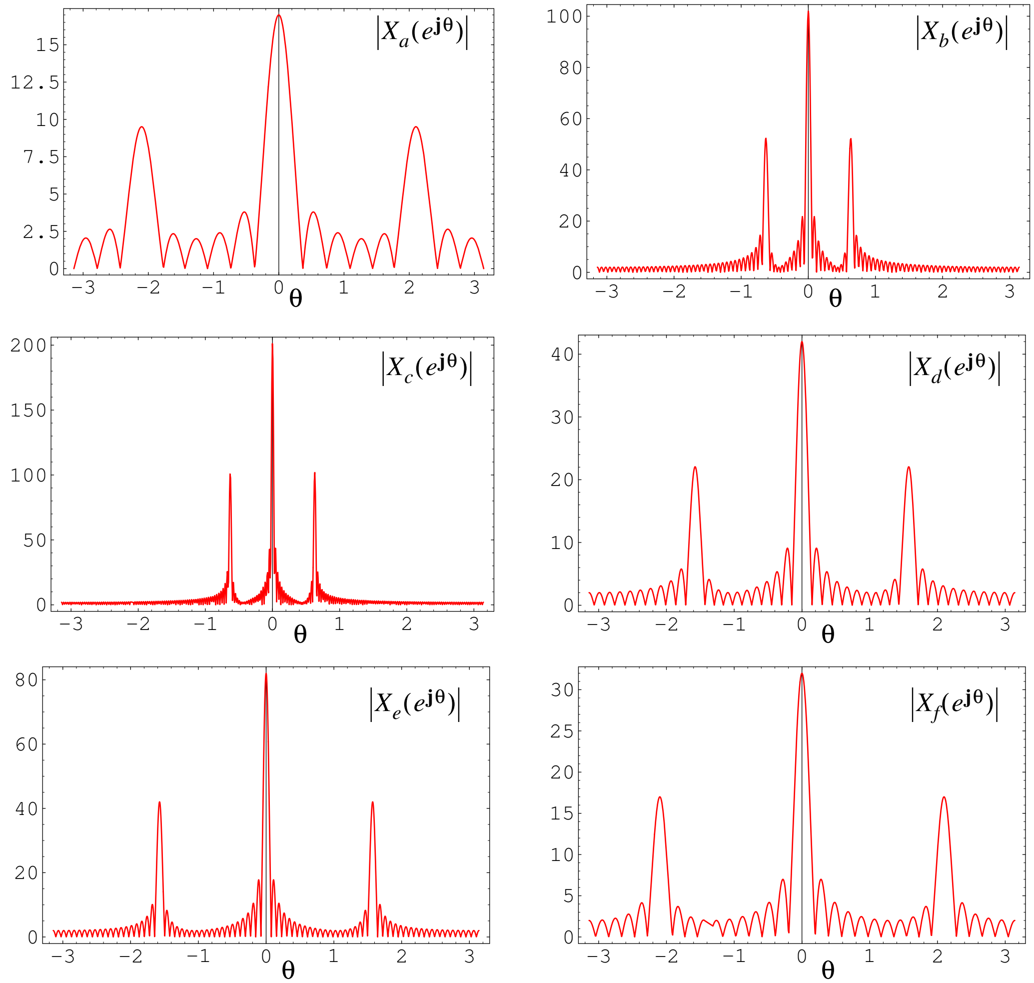
$$x_2[n] = x(n/3)(u[n] - u[n-30]),\ -\infty < n < \infty \quad \text{(13-2)}$$

$$x_3[n] = x(n/8)(u[n] - u[n-40]),\ -\infty < n < \infty \quad \text{(13-3)}$$

$$x_4[n] = x(n/8)(u[n] - u[n-80]),\ -\infty < n < \infty \quad \text{(13-4)}$$

$$x_5[n] = x(n/20)(u[n] - u[n-100]),\ -\infty < n < \infty \quad \text{(13-5)}$$

$$x_6[n] = x(n/20)(u[n] - u[n-200]),\ -\infty < n < \infty \quad \text{(13-6)}$$



where $x(t) = 1 + \cos(4\pi t)$, $-\infty < t < \infty$, to its corresponding magnitude DTFT representation below. Recall that $\theta = (2\pi f)/f_s$. Explain your answer.

$$x_1[n] \Leftrightarrow |X_a(e^{j\theta})|,\ x_2[n] \Leftrightarrow |X_f(e^{j\theta})|,\ x_3[n] \Leftrightarrow |X_d(e^{j\theta})|,\ x_4[n] \Leftrightarrow |X_e(e^{j\theta})|,\ x_5[n] \Leftrightarrow |X_b(e^{j\theta})|,$$
$$x_6[n] \Leftrightarrow |X_c(e^{j\theta})| \tag{S-60}$$

Explanation: Signals $x_1[n]$ and $x_2[n]$ are undersampled and should exhibit peaks at ±1 Hz and 0Hz [easily derivable through idealized aliasing analysis], corresponding to θ values,

$$\theta = (2\pi f)/f_s = \pm 2\pi/3, 0 \tag{S-61}$$

Signals $x_3[n]$ and $x_4[n]$ are sufficiently sampled and should exhibit peaks at ±2 Hz and 0Hz, corresponding to θ values,

$$\theta = (2\pi f)/f_s = \pm 4\pi/8, 0 = \pm\pi/2, 0 \tag{S-62}$$

Similarly, signals $x_5[n]$ and $x_6[n]$ are sufficiently sampled and should exhibit peaks at,

$$\theta = (2\pi f)/f_s = \pm 4\pi/20, 0 = \pm\pi/5, 0 \tag{S-63}$$

The above analysis helps us narrow down the potential matches for each time-domain signal to two. Exact matches between time-domain signals and frequency domain representations are made by realizing that longer-sampled sequences in the time-domain will result in more focused representations in the frequency domain.

Problem 14:

(a) Write a computer program (MATLAB, Mathematica, etc.) to plot the magnitude and phase (as a function of θ) of the DTFT for the following discrete-time signal:

$$x_1[n] = \begin{cases} x_c(n/10) & n \in \{0, 1, \ldots, 20\} \\ 0 & elsewhere \end{cases} \quad \text{where } x_c(t) = \cos(2\pi t) + 3\cos(4\pi t)\,. \tag{14-1}$$

(a) Now plot the magnitude and phase (as a function of θ) of the DTFT for the following discrete-time signal:

$$x_2[n] = x_1[n+10] \tag{14-2}$$

where $x_1[n]$ is given in (14-1) above.

See the plots below. Note that the shift in equation (14-2) only affects the phase part of the DTFT, not the magnitude part of the DTFT. (For MATLAB and Mathematica code, see the course web page.)