EEL3135: Homework #4

Problem 1:

For each of the systems below, determine whether or not the system is (1) linear, (2) time-invariant, and (3) causal:

(a) $y[n] = x[n]\cos(0.4\pi n)$

(b) $y[n] = x[n] - x[n-5]$

(c) $y[n] = x[|n|]$

(d) $y[n] = |x[n]|$

(e) $y[n] = 2x[n-2] + x[n+1]$

(f) $y[n] = 3x[n-2] + \sqrt{n}$

Problem 2:

Assume the impulse response of an LTI system is $h[n]$ given below:

$$h[n] = \delta[n+1] + 2\delta[n-2] - 3\delta[n-4] \tag{2-1}$$

(a) Give the difference equation for this LTI system.

(b) Compute the output $y[n]$ for an input of:

$$x[n] = 4\delta[n] + 2\delta[n-1] + 4\delta[n-4] - \delta[n-5] + 6\delta[n-6] \tag{2-2}$$

using the convolution operator for the range of n where $y[n] \neq 0$ (set up a table similar to one of the two tables in the notes).

(c) Verify your answer in part (b) by direct substitution into the difference equation for part (a).

Problem 3:

Assume that the response of an LTI system to the input $x[n]$,

$$x[n] = \delta[n] - \delta[n-1] \tag{3-1}$$

is given by,

$$y[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]. \tag{3-2}$$

For this system, compute the output $y_2[n]$ to the following input:

$$x_2[n] = 5\delta[n] - 5\delta[n-3]. \tag{3-3}$$

Problem 4:

Assume that the response of an LTI system to the input $x[n] = u[n]$ (discrete-time unit step) is given by,

$$y[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]. \tag{4-1}$$

(a) For this system, compute the output $y_2[n]$ to the following input:

$$x_2[n] = 3u[n] - 2u[n-4] \tag{4-2}$$

(b) Derive the impulse response $h[n]$ for this system.

(c) Give the difference equation for this system.

Problem 5:

Assume an LTI system of the following form:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] \qquad (5\text{-}1)$$

with impulse response,

$$h[n] = 3\delta[n] + 7\delta[n-1] + 13\delta[n-2] + 9\delta[n-3] + 5\delta[n-4] \qquad (5\text{-}2)$$

(a) Give numeric values for M and b_k, $k \in \{0, \ldots, M\}$.

(b) Compute $y[n]$, $\forall n$, for input $x[n]$ given by,

$$x[n] = \begin{cases} 0 & n = even \\ 1 & n = odd \end{cases} \qquad (5\text{-}3)$$

Problem 6:

For each part below, give an expression for $y[n]$. When applicable, assume $x[n] = 0$, $n < 0$; also, $u[n]$ denotes the discrete-time unit step function, and * denotes the convolution operator.

(a) $y[n] = u[n] * u[n]$

(b) $y[n] = x[n] * u[n]$

(c) $y[n] = u[n] * x[n] * \delta[n]$

(d) $y[n] = a^n u[n] * u[n]$, $|a| < 1$.

Problem 7:

(a) Compute the impulse response $h[n]$ for a filter with frequency response $H(e^{j\theta})$, as illustrated in Figure 1 below, using the inverse DTFT integral. Simplify your answer as much as possible.

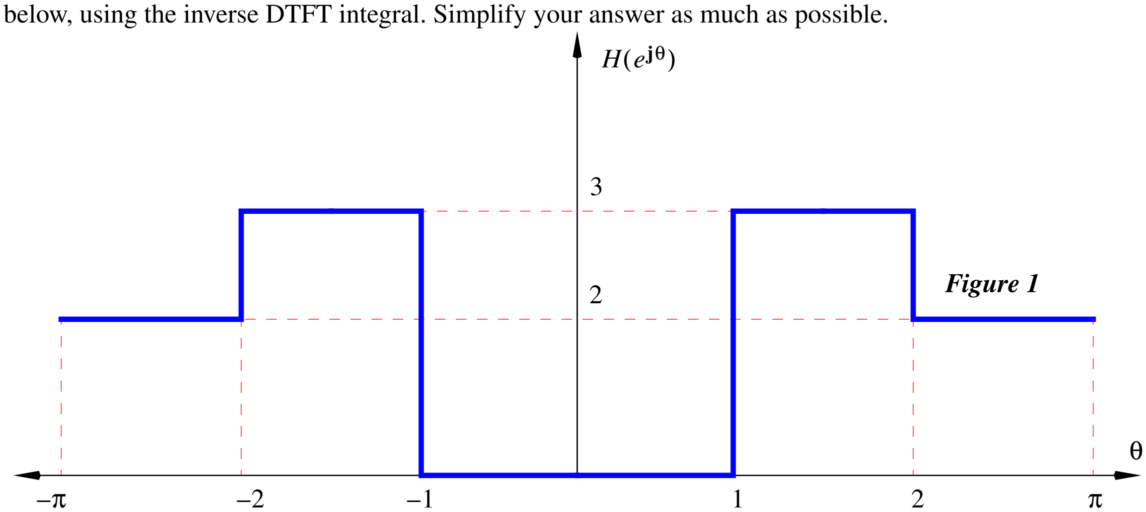

Figure 1

(b) Using L'Hopital's rule, simplify the formula for a general filter $h_g[n]$, derived in the notes, for $\Delta \to 0$, where,

$$h_g[n] = (1-a)\left[\frac{\cos(n(\theta_{c_1} - \Delta)) + \cos(n(\theta_{c_2} + \Delta)) - \cos(n\theta_{c_1}) - \cos(n\theta_{c_2})}{n^2 \pi \Delta}\right] + \delta[n] \qquad (7\text{-}1)$$

(c) Explain how you can use your result in part (b) to derive the impulse response $h[n]$ for part (a), without explicit computation of the inverse DTFT integral.

Problem 8:

Given an IIR filter defined by the difference equation:

$$y[n] = \left(-\frac{1}{2}\right)y[n-1] + x[n] \tag{8-1}$$

(a) Determine the transfer function $H(z)$ for this system.

(b) Compute the system poles.

(c) Compute $h[n]$ for this system. Is this system BIBO-stable?

(d) Determine $y[n]$ for $x[n] = \delta[n] - 2\delta[n-1] + 4\delta[n-4]$.

Problem 9:

Given an IIR filter defined by the difference equation:

$$y[n] = \sqrt{2}y[n-1] - y[n-2] + x[n] \tag{9-1}$$

(a) Determine the transfer function $H(z)$ for this system.

(b) Compute the system poles.

(c) Compute $h[n]$ for this system. Is this system BIBO-stable?

(d) Determine $y[n]$ for $x[n] = \delta[n] - 3\delta[n-1] + 2\delta[n-4]$.

Problem 10:

Determine the discrete-time signals $x_a[n]$ and $x_b[n]$, respectively, corresponding to the following z-transforms:

$$X_a(z) = \frac{1 - z^{-1}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}} \text{ [part (a)]} \tag{10-1}$$

$$X_b(z) = \frac{1 + z^{-1}}{1 - 0.1z^{-1} - 0.72z^{-2}} \text{ [part (b)]} \tag{10-2}$$

Problem 11:

An LTI system has the transfer function $H(z)$,

$$H(z) = 1 - 3z^{-2} - 4z^{-4} \tag{11-1}$$

The input to this system is given by,

$$x[n] = 20 - 20\delta[n] + 20\cos(0.5\pi n + \pi/4),\ -\infty < n < \infty. \tag{11-2}$$

Determine the output $y[n]$ of the system for all n.

Problem 12:

(a) True or False: For the discrete-time system $y[n] = a^n x[n-1]$ the output for an arbitrary input $x[n]$ is given by $y[n] = x[n] * h[n]$, where $h[n]$ is the impulse response of the system.

(b) True or False: For the discrete-time system $y[n] = y[n-1] + x[n+1]$ the output for an arbitrary input $x[n]$ is given by $y[n] = x[n] * h[n]$, where $h[n]$ is the impulse response of the system.

(c) True or False: For the discrete-time system $y[n] = 2y[n-1] + x[n]$, the frequency response of the system is given by,

$$H(e^{\mathbf{j}\theta}) = \frac{1}{1 - 2e^{-\mathbf{j}\theta}} \tag{12-1}$$

(d) True or False: For the discrete-time system $y[n] = (1/2)y[n-1] + x[n]$, the frequency response of the system is given by,

$$H(e^{\mathbf{j}\theta}) = \frac{1}{1 - (1/2)e^{-\mathbf{j}\theta}} \tag{12-2}$$

(e) True or False: An LTI system with frequency response $H(e^{\mathbf{j}\theta}) = u[\theta + 1] - u[\theta - 1]$, $\theta \in [-\pi, \pi]$, is noncausal.

(f) Give the difference equation for an LTI system with transfer function $H(z)$:

$$H(z) = \frac{1 + z^2 + 3z^4}{z^5} \tag{12-3}$$

(g) Give the difference equation for an LTI system with transfer function $H(z)$:

$$H(z) = \frac{z^2 + 1}{z^2 - z + 1} \tag{12-4}$$

Problem 13:

(a) Give the difference equation of an LTI system with frequency response:

$$H(e^{\mathbf{j}\theta}) = 1 + \cos(2\theta) + \cos(5\theta) \tag{13-1}$$

Hint: This problem, when approached correctly, does not involve a lot of computation.

(b) Is your answer in part (a) causal? If not, specify a difference equation for a causal LTI system with the same magnitude frequency response as in equation (13-1).

Problem 14:

Consider the following discrete-time system:

$$y[n] = \frac{1}{2}y[n-1] + 2x[n] \tag{14-1}$$

(a) Compute the transfer function $H(z)$ of this system.

(b) Is this system stable?

(c) Give an expression for the output $y[n]$ for the input $x[n] = u[n]$, assuming that $y[n] = 0$, $n < 0$.

(d) Compute the impulse response $h[n]$ of this system.

(e) Compute the frequency response $H(e^{\mathbf{j}\theta})$ of this system.

(f) Give an expression for the output $y[n]$ for the following input:

$$x[n] = 10 + 5\delta[n-3] + 2\sin(0.5\pi n + 1),\ -\infty < n < \infty. \tag{14-2}$$

Problem 15:

Consider the LTI system below:

$$y[n] = \frac{1}{2}x[n] - \frac{1}{2}x[n-2]. \tag{15-1}$$

(a) Compute the frequency response $H(e^{\mathbf{j}\theta})$ for this system.

(b) Give an analytic expression for $|H(e^{\mathbf{j}\theta})|$. Be sure to simplify your answer as much as possible.

(c) Give an analytic expression for $\angle H(e^{\mathbf{j}\theta})$. Be sure to simplify your answer as much as possible.

(d) Compute the output of this system for the following two inputs:

$$x_1[n] = \cos(\pi n), \ -\infty < n < \infty \tag{15-2}$$

$$x_2[n] = \cos(\pi n/2), \ -\infty < n < \infty. \tag{15-3}$$

Problem 16:

Assume that you apply the following input $x[n]$ to an LTI system:

$$x[n] = \cos(\pi n/2)u[n] \tag{16-1}$$

and observe the following output $y[n]$:

$$y[n] = 3(2)^{-n}u[n] + 2\cos(\pi n/2)u[n]. \tag{16-2}$$

(a) Using the z-transform pair,

$$a^n u[n] \quad \Leftrightarrow \quad \frac{1}{1 - az^{-1}} \text{ show that } X(z) = \frac{1}{1 + z^{-2}}. \tag{16-3}$$

(b) Compute the transfer function $H(z)$ for this LTI system.

(c) Compute the impulse response $h[n]$ for this LTI system.

(d) Specify the difference equation that describes this LTI system. Is this an FIR or an IIR system?

(e) For the input in equation (16-1), explain why,

$$y[n] = |H(e^{\mathbf{j}\theta})|_{\theta = \pi/2} \cos(\pi n/2 + \angle H(e^{\mathbf{j}\theta})|_{\theta = \pi/2}) \tag{16-4}$$

does <u>not</u> give the output for this LTI system.

(f) Specify $|H(e^{\mathbf{j}\theta})|_{\theta = \pi/2}$ and $\angle H(e^{\mathbf{j}\theta})|_{\theta = \pi/2}$. <u>Hint</u>: This part requires no computation.