EEL6667: Homework #2

(8 problems, distributed 9/30/2003, due 10/14/2003)

Instructions:

You may use any mathematical package (e.g. Mathematica, Maple, MathCad, matlab) to help you solve these problems, as long as you turn in a complete printout of your code and runtime output.

Problem 1:

(a) The arm with three degrees of freedom shown in Figure 1 is like the one in Example 3.3 (Craig) except that joint 1's axis is not parallel to the other two. Instead, there is a twist of 90 degrees in magnitude between axes 1 and 2. Derive link parameters and the kinematic equations for ${}^{0}_{3}T$. Note that no l_3 need be defined. [Craig, Exercise 3.3]

(b) Derive the inverse kinematics of the three-link manipulator in Figure 1. [Craig, Exercise 4.2]

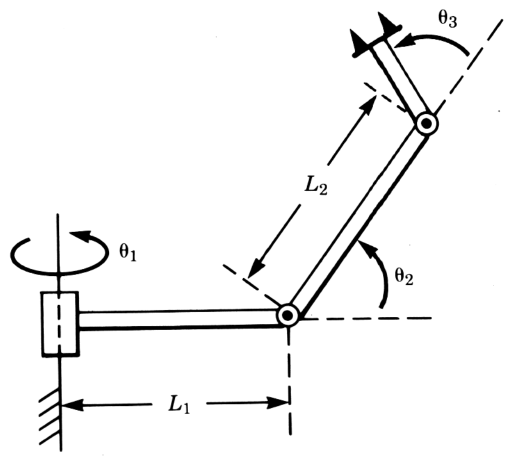

Figure 1

Problem 2:

(a) The arm with three degrees of freedom shown in Figure 2 has joints 1 and 2 perpendicular, and joints 2 and 3 parallel. As pictured, all joints are at their zero location. Note that the positive sense of the joint angles is indicated. Assign link frames $\{0\}$ through $\{3\}$ for this arm — that is, sketch the arm, showing the attachment of the frames. Then derive the transformation matrices ${}^{0}_{1}T$, ${}^{1}_{2}T$ and ${}^{2}_{3}T$. [Craig, Exercise 3.4]

(b) Derive the inverse kinematics of the 3-DOF manipulator in Figure 2. [Craig, Exercise 4.4]

Problem 3:[Craig, Exercise 3.14]

As was stated, the relative position of any two lines in space can be given with two parameters, a and α, where a is the length of the common perpendicular jointing the two, and α is the angle made by the two axes when projected onto a plane whose normal is the common perpendicular. Given a line defined as passing through point p with unit vector direction m, and a second passing through point q with unit vector direction n, give expressions for a and α.

Problem 4:[Craig, Exercise 3.16]

Assign link frames to the RPR planar robot shown in Figure 3 and give the linkage parameters.

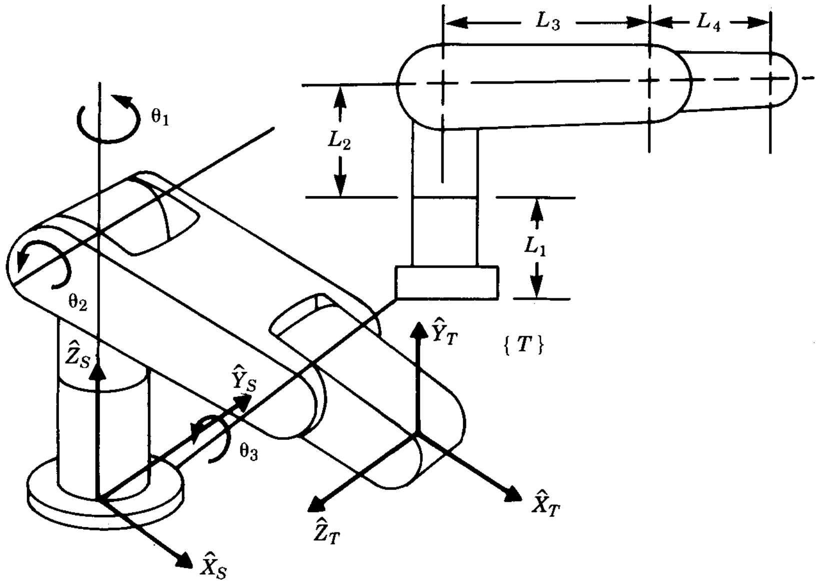

Figure 2

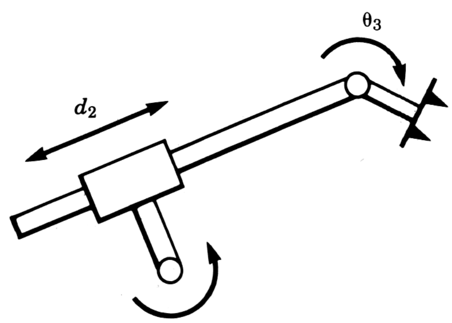

Figure 3

Problem 5:[Craig, Exercise 4.9]

Figure 4 shows a two-link planar arm with rotary joints. For this arm, the second link is half as long as the first, that is: $l_1 = 2l_2$. The joint range limits in radians are,

$$0 < \theta_1 < \pi \tag{1}$$

$$-\pi/2 < \theta_2 < \pi \tag{2}$$

Sketch the reachable workspace (an area) of the tip of link 2.

Problem 6:[Craig, Exercise 4.12]

In Figure 5, two 3R mechanisms are pictured. In both cases, the three axes intersect at a point (note that over all configurations, this point remains fixed in space). The mechanism in Figure 5(a) has link twists (α_i) of magnitude $\pi/2$. The mechanism in Figure 5(b) has one twist of ϕ in magnitude and the other of $(\pi - \phi)$ in magnitude.

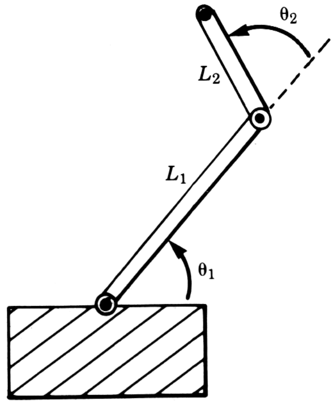

Figure 4

The mechanism in Figure 5(a) can be seen to be in correspondence with Z-Y-Z Euler angles, and therefore we know that it suffices to orient link 3 (with arrow in figure) arbitrarily with respect to the fixed link 0. Because ϕ is not equal to $\pi/2$, it turns out that the other mechanism cannot orient link 3 arbitrarily.

Describe the set of orentations which are unattainable with the second mechanism. Note that we assume that all joints can turn 2π radians (i.e. no limits) and we assume that the links may pass through each other if need be (i.e. workspace not limited by self-collisions).

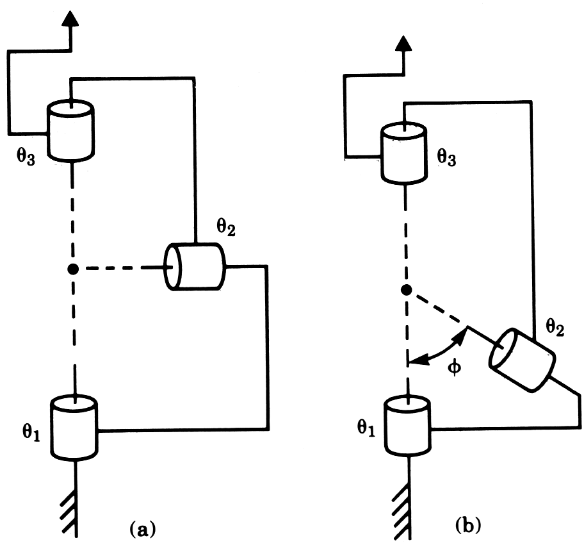

Figure 5

Problem 7:[Craig, Exercise 4.17]

A 4R manipulator is shown schematically in Figure 6. The nonzero link parameters are $\alpha_1 = -\pi/2$, $d_2 = 1$, $\alpha_2 = -\pi/4$, $d_3 = 1$ and $a_3 = 1$ and the mechanism is pictured in the configuration corresponding to,

$$\Theta = [0, 0, \pi/2, 0]^T. \tag{3}$$

Each joint has limits of $\pm\pi$. Find all values of θ_3 such that,

$$^0P_{4ORG} = [0, 1, \sqrt{2}]^T. \tag{4}$$

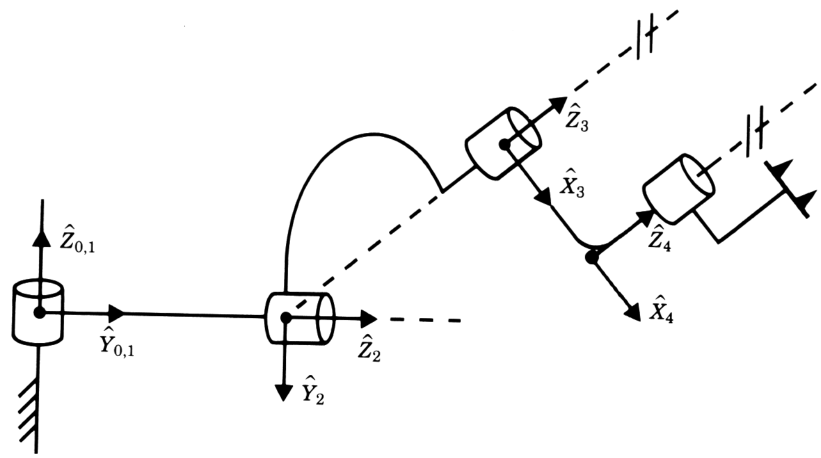

Figure 6

Problem 8:[Craig, Exercise 4.24]

Given the description of link frame $\{i\}$ in terms of link frame $\{i-1\}$, find the four Denavit-Hartenberg (DH) parameters as functions of the elements of ${}^{i-1}_{i}T$.