EEL6667: Homework #3

(9 problems, distributed 10/21/2003, due 11/6/2003)

Instructions:

You are strongly encouraged to use a mathematical package (e.g. Mathematica, Maple, MathCad, matlab) to help you solve these problems, as long as you turn in a complete printout of your code and runtime output.

Problem 1:[Craig, Exercise 5.2]

Find the Jacobian of the manipulator with three degrees of freedom in Figure 1 [Craig, Exercise 3.3]. Write it in terms of a frame $\{4\}$ located at the tip of the hand with the same orientation as frame $\{3\}$.

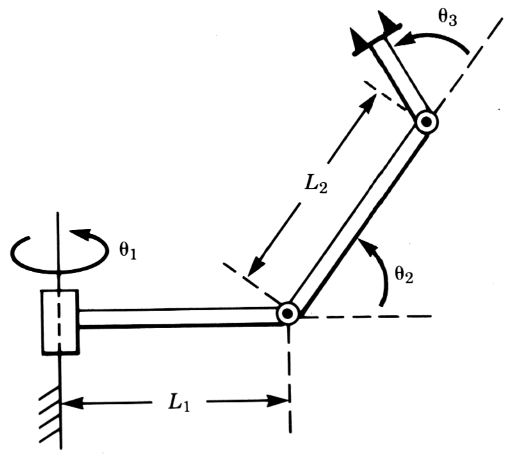

Figure 1

Problem 2:[Craig, Exercise 5.8]

General mechanisms may have certain configurations, called "isotropic points" where the columns of the Jacobian become orthogonal and of equal magnitude. For the two-link manipulator in Figure 2 [Craig, Example 5.3], determined if any isotropic points exist. Hint: Is there a requirement on L_1 and L_2?

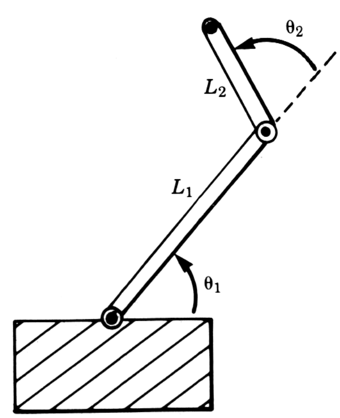

Figure 2

Problem 3:[Craig, Exercise 5.10]

For the two-link manipulator of Figure 2 [Craig, Example 5.3], give the transformation which would map joint torques into a 2×1 force vector, ${}^{3}F$, at the hand.

Problem 4:[Craig, Exercise 5.11]

Given

$$ {}^{A}_{B}T = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 10 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

if the velocity vector at the origin of $\{A\}$ is

$$ {}^{A}\upsilon = \begin{bmatrix} 0 \\ 2 \\ -3 \\ \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix} \tag{2} $$

find the 6×1 velocity vector with reference point at the origin of $\{B\}$.

Problem 5:[Craig, Exercise 5.15]

Give the 3×3 Jacobian which calculates linear velocity of the tool tip from the three joint rates for the manipulator in Figure 3 [Craig, Example 3.4]. Give the Jacobian in frame $\{0\}$.

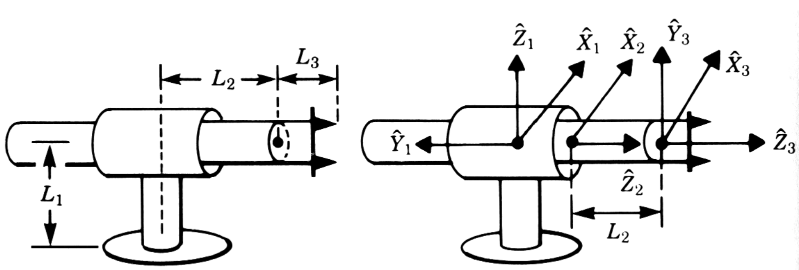

Figure 3

DH parameters for Figure 3

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	0	d_2	0
3	0	0	L_2	θ_3
4	0	0	L_3	0

Problem 6:[Craig, Exercise 5.16]

A $3R$ manipulator has kinematics that correspond exactly to the set of $Z-Y-Z$ Euler angles (i.e. the forward kinematics are given by,

$$R_{ZYZ}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & c\beta \end{bmatrix} \tag{3}$$

with $\alpha = \theta_1$, $\beta = \theta_2$ and $\gamma = \theta_3$). Give the Jacobian relating joint velocities to angular velocity of the final link.

Problem 7:[Craig, Exercise 5.17]

Imagine that for a general 6-DOF robot we have available ${}^0\hat{Z}_i$ and ${}^0P_{iorg}$ for all i. That is, we know the values for the unit Z vectors of each link frame in terms of the base frame, and also the locations of the origin of all link frames in terms of the base frame. Let us say that we are interested in the velocity of the tool point (fixed relative to link n) and we know ${}^0P_{tool}$ also. Now, for a revolute joint, the velocity of the tool tip due to the velocity of joint i is given by

$${}^0v_i = \dot{\theta}_i\,{}^0\hat{Z}_i \times ({}^0P_{tool} - {}^0P_{iorg}), \tag{4}$$

and the angular velocity of link n due to the velocity of this joint is given by

$${}^0\omega_i = \dot{\theta}_i\,{}^0\hat{Z}_i \tag{5}$$

The total linear and angular velocity of the tool is given by the sum of the ${}^0v_i$ and ${}^0\omega_i$ vectors, respectively. Give equations analogous to (4) and (5) for the case of joint i prismatic, and write the 6×6 Jacobian matrix of an arbitrary 6-DOF manipulator in terms of the $\hat{Z}_i$, P_{iorg} and P_{tool} vectors.

Problem 8:[Craig, Exercise 5.18]

The kinematics of a $3R$ robot are given by

$${}^0_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & l_1c_1 + l_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & l_1s_1 + l_2s_1c_2 \\ s_{23} & c_{23} & 0 & l_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Find ${}^0J(\Theta)$ which, when multiplied by the joint velocity vector, gives the linear velocity of the origin of frame $\{3\}$ relative to frame $\{0\}$.

Problem 9:[Craig, Exercise 5.19]

The position of the origin of link 2 for an RP manipulator is given by

$${}^0P_{2ORG} = \begin{bmatrix} a_1c_1 - d_2s_1 \\ a_1s_1 + d_2c_1 \\ 0 \end{bmatrix} \tag{7}$$

Give the 2×2 Jacobian that relates the two joint rates to the linear velocity of the origin of frame $\{2\}$. Give a value of Θ where the device is at a singularity.