EEL6667: Homework #4

*(6 problems, distributed 12/04/2003, due 12/20/2003, midnight, ** problems = extra credit)*

Instructions:

You are strongly encouraged to use a mathematical package (e.g. Mathematica, Maple, MathCad, matlab) to help you solve these problems, as long as you turn in a complete printout of your code and runtime output.

Problem 1:**

(a) Derive the dynamic model of the three-link manipulator in Figure 1. Specify your answer in terms of $M(\Theta)$, $V(\Theta, \dot{\Theta})$ and $G(\Theta)$ where,

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta) . \quad (1)$$

Assume that each link is composed of a thin, uniform-density rod of mass m_i, $i \in \{1, 2, 3\}$, that the length of link 3 is given by L_3, and let the gravitational acceleration be denoted by g.

(b) Now, assume that $L_1 = L_2 = L_3 = L$ and $m_1 = m_2 = m_3 = m$. Show that for these parameters, the mass matrix $M(\Theta)$ is positive definite for $L_i, m_i > 0$, $i \in \{1, 2, 3\}$. For this problem, it is sufficient to show that,

$$\det[M(\Theta)] > 0 , \forall\Theta . \quad (2)$$

(c) Suppose that the manipulator is holding a point-mass object of mass m_o. What are the new dynamics associated with link 3? In your answer, it is sufficient to give the new dynamic parameters,

$$\left\{ {}^{C_{3'}}I_{3'}, m_{3'}, {}^{3'}P_{C_{3'}} \right\} \quad (3)$$

of the manipulator/object system, assuming that link $3'$ is modeled as the combination of link 3 and the point-mass object.

Note: For parts (a) and (b), the *Mathematica* notebook,

```
http://mil.ufl.edu/~nechyba/eel6667/assignments/hw4/hw4_template.nb
```

might serve as a useful template. Note that the above notebook requires that the definitions in the following notebook be evaluated first:

```
http://mil.ufl.edu/~nechyba/eel6667/assignments/hw4/manipulator_defs.nb
```

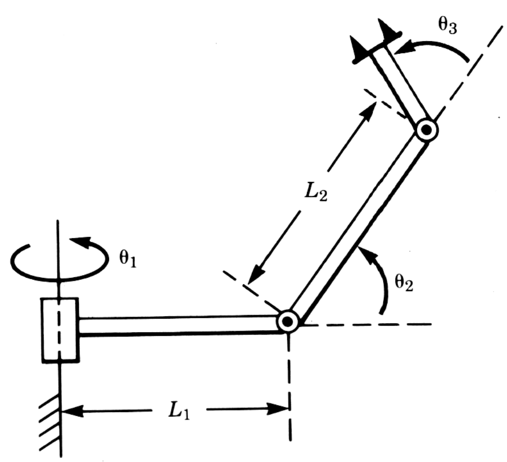

Figure 1

Problem 2:[Craig, Exercise 6.14]

The equations below were derived for a 2-DOF RP manipulator. However, some of the terms are obviously incorrect. Indicate the incorrect terms, and explain why they are incorrect.

$$\tau_1 = m_1(d_1^2 + d_2)\ddot{\theta}_1 + m_2 d_2^2 \ddot{\theta}_1 + 2 m_2 d_2 \dot{d}_2 \dot{\theta}_1 + g\cos(\theta_1)[m_1(d_1 + d_2\dot{\theta}_1) + m_2(d_2 + \dot{d}_2)] \quad (4)$$

$$\tau_2 = m_1 \dot{d}_2 \ddot{\theta}_1 + m_2 \ddot{d}_2 - m_1 d_1 \dot{d}_2 - m_2 d_2 \dot{\theta}^2 + m_2(d_2 + 1) g \sin(\theta_1) \quad (5)$$

Problem 3:**

Figure 2 shows two views of a uniform-density body composed of three separate parts D, E and F. Part E is a solid cylinder of radius r and length l. Parts D and F are holed cylinders with inner radius a, outer radius b and height h. Compute the mass moments of inertia ${}^C I_{xx}$, ${}^C I_{yy}$ and ${}^C I_{zz}$. [Note that parts D and E are rotated 90 degrees with respect to one another, and that coordinate frame $\{C\}$ is fixed to the geometric center of cylinder E.]

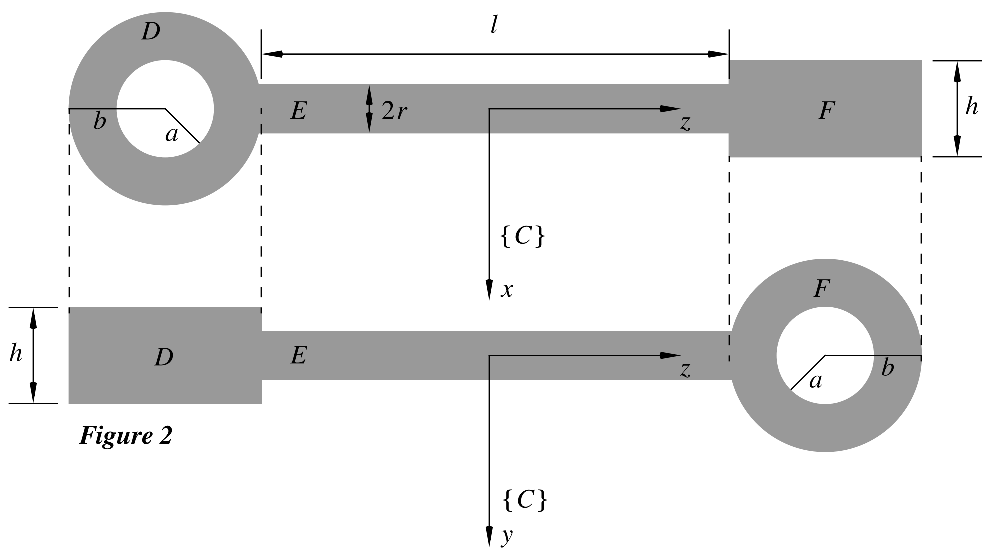

Figure 2

Problem 4:

Suppose you want to construct a one-dimensional trajectory $\{t, x\}$ composed of two cubic path segments, with starting point $\{t_o, x_o\} = \{0, 5\}$, via point $\{t_v, x_v\} = \{2, 10\}$ and ending point $\{t_f, x_f\} = \{3, 0\}$.

(a) Compute the coefficients of the two cubic polynomials for this trajectory, enforcing continuous acceleration of your trajectory at $t_v = 2$. Plot x, $\dot{x}$ and $\ddot{x}$ versus time t.

(b) Compute the coefficients of the two cubic polynomials for this trajectory, enforcing zero velocity of your trajectory at $t_v = 2$. Plot x, $\dot{x}$ and $\ddot{x}$ versus time t.

(c) Which of these two trajectories is not possible if x is not allowed to be greater than 11?

(d) Which of these two trajectories exhibits larger accelerations $\ddot{x}$?

Problem 5:

Let a serial-link manipulator be mounted on a satellite platform in space as shown in Figure 3. When the manipulator is moving, describe what force/torque vector the thrusters of the satellite must apply to the satellite/manipulator system in order to keep the satellite in equilibrium. Assume the kinematic and dynamic parameters of the manipulator are completely known.

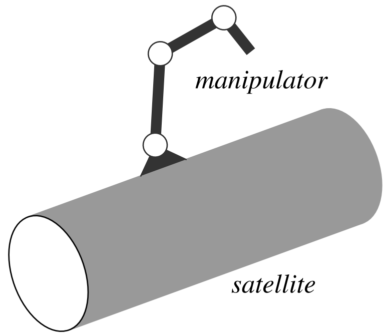

Figure 3

Problem 6:

In this problem, you will experiment with different controllers for a four-link manipulator, whose DH parameters are given in the table below.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	0	0	θ_2
3	0	L_1	0	θ_3
4	0	L_2	0	θ_4
5	0	L_3	0	0

Essentially, this manipulator consists of a two-DOF wrist at the base, followed by three links of non-zero length. The masses m_i, $i \in \{1, 2, 3, 4\}$, of each link is concentrated as a point mass at the distal end of that link; consequently,

$$^{1}P_{C_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ ^{i}P_{C_i} = \begin{bmatrix} L_{i-1} \\ 0 \\ 0 \end{bmatrix}, \ i \in \{2, 3, 4\}, \tag{6}$$

$$^{C_i}I_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ i \in \{1, 2, 3, 4\}. \tag{7}$$

Now, ideally the robot is oriented with respect to the world such that,

$$^{0}(\dot{v}_0)_{ideal} = g\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T. \tag{8}$$

However, the robot is slightly tilted, so that in reality,

$$^{0}(\dot{v}_0)_{actual} = g\left[k_1\ k_2\ k_3\right]^T \tag{9}$$

where,

$$k_1 = k_2 = \frac{1}{\sqrt{9803}},\ k_3 = \frac{99}{\sqrt{9803}} \text{ and } g = 9.81\ m/s^2. \tag{10}$$

The numeric values for the link lengths are,

$$L_1 = 1.000,\ L_2 = 0.750 \text{ and } L_3 = 0.500 \text{ (all values in meters)} \tag{11}$$

while the real values for the masses are,

$$m_1 = 1.100,\ m_2 = 1.800,\ m_3 = 1.575 \text{ and } m_4 = 1.100 \text{ (all values in kg's).} \tag{12}$$

However, in our model of the manipulator, we use the following values for the masses:

$$m_1 = 1.000,\ m_2 = 2.000,\ m_3 = 1.500 \text{ and } m_4 = 1.000 \text{ (all values in kg's).} \tag{13}$$

and the idealized base acceleration vector in equation (8) instead of the actual base acceleration vector in equation (9). In order to help you conduct these experiments, a control simulator for this manipulator has been written in C. The code files are located in the directory:

```
http://mil.ufl.edu/~nechyba/eel6667/assignments/hw4/manip
```

and are listed below:

```
Makefile
control.c
control.h
manip.c
manip.h
manip_main.c
robots.h
2link_cylindrical.c
2link_pointmass.c
4link_pointmass.c
```

The `Makefile` executes the following commands (on a Unix/Linux system):

```
gcc -O3 -I. -Wall   -c -o manip_main.o manip_main.c
gcc -O3 -I. -Wall   -c -o manip.o manip.c
gcc -O3 -I. -Wall   -c -o control.o control.c
gcc -O3 -I. -Wall   -c -o 2link_pointmass.o 2link_pointmass.c
gcc -O3 -I. -Wall   -c -o 2link_cylindrical.o 2link_cylindrical.c
gcc -O3 -I. -Wall   -c -o 4link_pointmass.o 4link_pointmass.c
gcc -O3 -I. -Wall -o msim manip_main.o manip.o control.o 2link_pointmass.o
       2link_cylindrical.o 4link_pointmass.o -lm
/bin/rm -f *.o
```

Now the executable `msim` can be run as follows:

```
msim <parameter_file> <output_file>
```

You can find a sample parameter file with explanations at:

```
http://mil.ufl.edu/~nechyba/eel6667/assignments/hw4/sample.params
```

The output file has 13 columns, where each line consists of:

$$t \quad \theta_{1d}(t) \quad \theta_{2d}(t) \quad \theta_{3d}(t) \quad \theta_{4d}(t) \quad \theta_{1a}(t) \quad \theta_{2a}(t) \quad \theta_{3a}(t) \quad \theta_{4a}(t) \quad \tau_1(t) \quad \tau_2(t) \quad \tau_3(t) \quad \tau_4(t)$$

where $\theta_{id}(t)$ denotes the desired value in radians of joint i at time t, $\theta_{ia}(t)$ denotes the actual (controlled) value in radians of joint i at time t, and $\tau_i(t)$ denotes the torque in $N \cdot m$ applied at joint i at time t.

You can plot all the trajectories for a particular controller using the following *Mathematica* notebook:

```
http://mil.ufl.edu/~nechyba/eel6667/assignments/hw4/visualization.nb
```

Alternatively, you can use matlab (or any other tool), or for quick plots (in Unix/Linux) you can run `gnuplot`. In `gnuplot`, you can, for example, plot $\theta_{1a}(t)$ using the following command:

```
plot "results.dat" using 1:6 with lines
```

since $\theta_{1a}(t)$ is the sixth column of the output file (`results.dat`, in this case).

Now, suppose you want the four-link robot to move from an initial joint configuration Θ_{start} (parameter: `Start`) to a final joint configuration Θ_{goal} (parameter: `Goal`) along a cubic trajectory where,

$$\Theta_{start} = \begin{bmatrix} 0 \\ 45° \\ -90° \\ -135° \end{bmatrix}, \Theta_{goal} = \begin{bmatrix} 90° \\ 90° \\ 0° \\ -90° \end{bmatrix}, \tag{14}$$

and the start and stop times are given by,

$$t_{start} = 0 \text{ (sec) and } t_{goal} = 10 \text{ (sec)}, \tag{15}$$

and $\|\tau_{max}\| = 50\ N \cdot m$. Finally, let the control frequency (parameter: `Frequency`) be 100 Hz.

(a) Consider the following PID control law (parameter: `Controller 2`),

$$\tau = \ddot{\Theta}_d + K_v\dot{E} + K_pE + K_i\int E dt. \tag{16}$$

Using this control law, can you find a set of gains K_v (parameter: `KV`), K_p (parameter: `KP`) and K_i (parameter: `KI`), which result in torques that stay within the required torque limit $\|\tau_{max}\|$, and a controlled trajectory that approximately follows the above-specified cubic trajectory and settles to less than 0.01 rad absolute error/joint within 5 seconds of reaching t_{goal}? Report your results. In your results, include your parameter file, and plots of the desired trajectory, actual trajectory and applied torques.

(b) Consider the following gravity-compensated PID control law (parameter: `Controller 4`),

$$\tau = \ddot{\Theta}_d + K_v\dot{E} + K_pE + K_i\int E dt + \hat{G}(\Theta). \tag{17}$$

Using this control law, determine a set of gains K_v (parameter: `KV`), K_p (parameter: `KP`) and K_i (parameter: `KI`), which result in torques that stay within the required torque limit $\|\tau_{max}\|$, and a controlled trajectory that approximately follows the above-specified cubic trajectory and settles to less than 0.01 rad absolute error/joint within 5 seconds of reaching t_{goal}. Report your results. In your results, include your parameter file, and plots of the desired trajectory, actual trajectory and applied torques.

(c) Consider the following partitioned control law (parameter: `Controller 6`),

$$\tau = \alpha\tau' + \beta \tag{18}$$

$$\alpha = \hat{M}(\Theta) \tag{19}$$

$$\beta = \hat{V}(\Theta, \dot{\Theta}) + \hat{G}(\Theta) \tag{20}$$

$$\tau' = \ddot{\Theta}_d + K_v\dot{E} + K_pE + K_i\int E dt. \tag{21}$$

Using this control law, determine a set of gains K_v (parameter: `KV`), K_p (parameter: `KP`) and K_i (parameter: `KI`), which result in torques that stay within the required torque limit $\|\tau_{max}\|$, and a controlled trajectory that approximately follows the above-specified cubic trajectory and settles to less than 0.01 rad absolute error/joint within 5 seconds of reaching t_{goal}. Report your results. In your results, include your parameter file, and plots of the desired trajectory, actual trajectory and applied torques.

(d) Compare the three controllers in parts (a), (b) and (c) in terms of performance and maximum required torques.